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AN ATTEMPT

TO TEST THE THEORIES OF CAPILLARY ACTION.

London: C. J. CLAY, M.A. & SON,
CAMBRIDGE UNIVERSITY PRESS WAREHOUSE,
17, PATERNOSTER ROW.

CAMBRIDGE: DEIGHTON, BELL, AND CO.
LEIPZIG: F. A. BROCKHAUS.

AN ATTEMPT

TO TEST

THE THEORIES OF CAPILLARY ACTION

BY COMPARING

THE THEORETICAL AND MEASURED FORMS OF DROPS OF FLUID,

BY

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WITH

AN EXPLANATION OF THE METHOD OF INTEGRATION
EMPLOYED IN CONSTRUCTING THE TABLES WHICH GIVE THE THEORETICAL
FORMS OF SUCH DROPS,

BY

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Cambridge:
AT THE UNIVERSITY PRESS.
1883

The writers take this opportunity of returning their thanks to the Syndics of the University Press for undertaking the expense of printing this work.

INTRODUCTION.

MANY years have elapsed since this work was commenced, and it is even now only partially completed. My object was to test the received theories of Capillary Action, and through them the assumed laws of molecular attraction, on which they are founded. To this end it was proposed to compare the actual forms of drops of fluid resting on horizontal planes they do not wet, with their theoretical forms.

After some trials a satisfactory micrometrical instrument was constructed for the measurement of the forms of drops of fluid, but my attempts to calculate their forms as surfaces of *double* curvature failed entirely, and my undertaking must have ended here, if I had depended upon my own resources. But at this point Professor J. C. Adams furnished me with a perfectly satisfactory method of calculating by quadratures the exact theoretical forms of drops of fluids from the Differential Equation of Laplace, an account of which he has now had the kindness to prepare for publication. After the calculation of a few forms, application was made to the Royal Society for assistance from the Government grant in making the needful calculations. The following extracts from the application (Oct. 27, 1855) will explain the objects of the undertaking. "I have carefully examined all the published "experiments that I could meet with, but these have been generally made with "capillary tubes, and in consequence of the difficulties inherent in this mode of "observation they have not led to consistent and satisfactory results.

"It appeared to me that the best test of theory would be obtained by making "careful measures of the forms assumed by drops of fluid resting on horizontal "planes of various solids......"

"At first I knew of no better mode of arriving at the theoretical forms than "that given by geometrical construction, but I am indebted to Mr Adams for a "method of treating the differential equation

$$\frac{\dfrac{ddz}{du^2}}{\left\{1+\dfrac{dz^2}{du^2}\right\}^{\frac{3}{2}}}+\frac{\dfrac{1}{u}\dfrac{dz}{du}}{\left(1+\dfrac{dz^2}{du^2}\right)^{\frac{1}{2}}}-2\alpha z=\frac{2}{b},$$

"when put under the form $\dfrac{b}{\rho}+\dfrac{b}{x}\sin\phi=2+2\alpha b^2\dfrac{z}{b}=2+\beta\dfrac{z}{b}$,

"which gives the theoretical form of the drop with an accuracy exceeding that of "the most refined measurements. Values of $\frac{x}{b}$, $\frac{z}{b}$ and $\frac{\rho}{b}$ have been calculated by this "method for values of ϕ at intervals of $2\frac{1}{2}^\circ$ to 5°, from $\phi = 0$ to $\phi = 145^\circ$, for "values of β equal to $\frac{1}{4}$, $\frac{1}{2}$, 1, 3, 6, 10 and 16. It is however very desirable that "calculations should be made for more numerous as well as for larger values of β.

"I also propose to make accurate measurements of the forms of the common "surfaces of two fluids that do not mix. The form of a drop of fluid (A) will "be taken when immersed in a fluid (B), and also the form of a drop of the fluid "(B) when immersed in fluid (A), and for this purpose a plate-glass cell has been "constructed, so that the observations can be made whether the drops rest on the "bottom, or float in contact with the upper surface. The forms of drops of fluids "(A) and (B) will also be taken when resting on horizontal planes surrounded by the "atmosphere."

"The objects of the experiments are

"I. To compare the actual forms assumed by drops of fluid when resting on "horizontal planes composed of substances which they do not wet, with their theo-"retical forms.

"II. To determine the effects of supporting planes composed of various sub-"stances.

"III. To examine the effects of different degrees of roughness of the supporting "planes composed of various substances.

"IV. To determine the effects of variations of temperature on the forms of the "drops of fluid from 32° to about 200° F.

"V. To examine the mutual action of two fluids that do not mix, and the "effects of variation of temperature on them."

The Royal Society voted a grant of £50, the sum applied for. These calculations were completed in 1857. And after the calculation of the theoretical forms and volumes of *sessile* drops had been carried as far as seemed needful, the money in hand was applied to the calculation of theoretical forms and volumes of *pendent* drops of fluids. The results of these calculations have been printed in Table IV.

The delay in the publication of my results has arisen from the interruption of my labours, caused first by my removal in 1857 from College to a country living, and secondly by my appointment in 1864 to the Professorship of Applied Mathematics to the Advanced Class of Royal Artillery Officers, Woolwich. As no systematic experiments had then been made since the time of Hutton to determine the Resistance of the Air to the motion of projectiles, and those for round shot only, I was induced to turn my attention to the subject of Ballistics. The Results of my Experiments have been published under the authority of the Secretary of State for War, as follows—

I. Reports on Experiments made with the Bashforth Chronograph to determine the Resistance of the Air to the Motion of Projectiles, 1865—1870. London, W. Clowes and Sons, &c. &c.

II. Final Report on Experiments, &c. &c., 1878—80. London, W. Clowes and Sons, &c. &c.

And in connection with these Reports I published a Mathematical Treatise on the Motion of Projectiles, 1873, and a Supplement to that work, 1881.

Immediately after the completion of these labours I turned my attention to the preparation for publication of a part of my work on Capillary Action, for I cannot now hope to be able to complete the work originally proposed. The Tables II. and III. appear to give all that is required in order to supply the means for filling up the intervals to five places of decimals for all values of β under 100, and of ϕ under 180°. The Table IV. for negative values of β, although not so complete, will afford considerable assistance, and the deficiencies can be easily supplied by original calculation preparatory to interpolation.

Table V. gives the theoretical forms of free capillary surfaces of revolution about a vertical axis, which was used in calculating the forms of drops of mercury shewn in the diagrams. Deficiencies may be easily supplied by the help of Table II. by interpolation.

As a specimen of the work I proposed to do, I have given diagrams and coordinates observed and calculated of forms of drops of mercury carefully measured in 1863. These shew how correctly the calculated and measured forms of these drops agree, notwithstanding the very considerable variation in their outlines.

Also, as I found my measuring instrument in good working order in 1882, I have made numerous measurements of drops of the same kind of mercury of 4, 8, 12, 16, 20 and 24 grs. in order to find the values of α and ω. The values derived from each particular measurement vary considerably—but the mean results for each weight of drop are satisfactory and appear to confirm the received theories of Capillary Action. But as the Theories of Young, Laplace, Gauss and Poisson lead to the same differential equation, and therefore give the same form of drops of fluid, experiments of this kind are not capable of deciding whether Poisson is correct in supposing that a rapid change of density takes place near the free surfaces of fluids. But more definite information on this head may be expected when the values of α and ω at the common surfaces of fluids which do not mix, as well as the effect of variation of temperature on these quantities, have been determined according to the original scheme.

Having given examples of the work I proposed to myself in the first instance, I must leave to others the further examination of this important question, for it still appears to me that this is the only way by which we can arrive at any definite results.

I take this opportunity to return my best thanks to the Syndics of the University Press for having undertaken the publication of this work.

CHAPTER I.

THEORETICAL EXPLANATIONS OF CAPILLARY ACTION.

THE phenomena which arise from Capillary Action seem to contradict the laws of fluid equilibrium. In consequence, many worthless theories have been proposed with a view to explain apparent anomalies. After long groping in the dark, it was found to be desirable to discover by experiment what were the actual phenomena which required explanation. Hawksbee[a] found that the height to which a fluid would rise in a capillary tube of given radius was the same for all thicknesses of the tube. From this it was apparent that the attracting force of the tube was situated at or near the inner surface of the tube. But he does not appear to have taken account of the mutual attractions of the particles of the fluid. Jurin[b] also found that the height of the column of fluid supported by capillary action depended solely upon the interior diameter of the tube at the upper surface of the fluid. From this he concluded that the column of fluid raised by Capillary Action was supported by the attraction of the periphery or section of the tube to which the upper surface of the fluid cohered or was contiguous.

Clairaut[c] was the first to attempt to explain capillary phenomena on right principles, by referring them to the mutual attraction of the particles of the fluid, and to the attraction of the particles of the solid on the particles of the fluid; and supposing these attractions to depend upon the same function of the distance, he concludes that even if the attraction of the capillary tube be of a less intensity than that of the water, provided the intensity of the latter attraction be not twice as great as that of the former, the water will still rise in the tube (p. 121). Clairaut supposed that the attraction was sensible only at very small distances (p. 113).

Shortly afterwards Segner[d] introduced the supposition that forces of attraction of both the particles of the solid and of the fluid vanished at sensible distances. He concluded that these forces gave a constant tension to the free capillary surfaces, and

[a] *Phil. Trans.*, 1711 and 1712.
[b] *Ibid.* 1718 and 1719.
[c] *Théorie de la Figure de la Terre*, 1743. Chapitre x.
[d] *Commentarii Soc. Reg. Sci. Gottingensis.* T. 1, 1751.

thence he tried to calculate the forms of sessile drops of fluid with a view to compare them with their measured forms. But in his calculations he took into account only the curvature of the vertical sections made by a plane passing through the axis of the drop. His measurements of the actual forms appear not to have been very precise.

An important paper on the Cohesion of Fluids was read before the Royal Society by Dr Young[a] in which he pointed out the necessity of taking into account the curvatures of both of the principal sections of the drop, and clearly propounded the true principles on which the solution of the problem must depend. He arrived at the conclusions (1) that the tension of a free capillary surface would be constant, and (2) that the angle of contact between a given solid and fluid surface would also be constant. He attempted to derive these hypotheses from physical considerations, but it is not easy to follow his reasoning. Even the editor of his works, Dean Peacock, observes on his *Analysis of the Simplest Forms* that "In the original Essay, the "mathematical form of this investigation and the figures were suppressed, the reasoning "and the results to which it leads being expressed in ordinary language: even in its "altered form the investigation is unduly concise and obscure"[b]. And respecting the appropriate angle of contact, Young confesses that "the whole of this reasoning on the "attraction of solids is to be considered rather as an approximation than as a strict "demonstration"[c]. This may in part be urged as a reason why Laplace[d] did not more fully recognise the value of Young's labours. And although many of their results agreed, the processes by which they arrived at them were very different, except that they were much on a par in respect to the constancy of the angle of contact, which Laplace did not deduce mathematically from his theory. Very good accounts of Laplace's Theory were given by Petit[e] and Pessuti[f], while it was attacked by others, as Young[g], Brunacci[h], Poisson[i] and others.

Gauss[k] by a new and striking mathematical investigation obtained the same differential equation to the form of capillary surfaces as Laplace had done, and also supplied the defect of his work by obtaining an expression for the angle of contact of the fluid with the solid. Like Laplace he supposed the fluid to be homogeneous and incompressible. Bertrand[l] has published a Memoir on Capillary Action, with a view to make known the method of Gauss, as well as some simplifications of which it is susceptible.

In 1831, Poisson published his important work, the *Nouvelle Théorie de l'Action Capillaire.* He strongly objects to Laplace's Theory because he has omitted in his calculations to take account of a physical circumstance, the consideration of which was essential; that is, the rapid variation of density which the liquid suffers near

[a] Dec. 20, 1804.

[b] *Works*, Vol. I., p. 420 (note).

[c] *Ib.* p. 434.

[d] *Méc. Cél. Supp. au X Livre*, 1806, 1807.

[e] *Journal de l'école Polytechnique.* Cahier XVI, 1813.

[f] *Mem. Soc. Ital.* T. XIV.

[g] *Quarterly Review and Works*, Vol. I., p. 436.

[h] *Brugnatelli*, T. IX., 1816.

[i] *Nouvelle Théorie*, 1831.

[k] *Princip. Gen. Theo. Fig. Fluid.* Gott. 1830. *Dove's Repertorium*, Bd. V., p. 49.

[l] *Liouville* XIII., p. 185.

its free surface, and near the solid against which it rests, "sans laquelle les phéno- "mènes capillaires n'auraient pas lieu"[a]. But he, in fact, arrives at a differential equation of precisely the same form as Young, Laplace and Gauss. It must be confessed that Poisson is probably quite right in supposing a rapid variation of density near the free surface of a fluid, and he has done good service in shewing how this supposed variation of density near the free surface of fluids may be taken account of in the mathematical treatment of Capillary Action. The reader may be further referred to a *Mémoire sur la Théorie de l'Action Moléculaire*, par Jean Plana[b].

[a] *Nouvelle Théorie*, p. 5.

[b] *Turin Mémoires*, 2 Série, T. XIV.

CHAPTER II.

EXPERIMENTAL TESTS OF THEORIES OF CAPILLARY ACTION.

MANY attempts have been made in recent times to test by experiment these theoretical explanations of capillary phenomena. For this purpose Haüy and Tremery[a] at the request of Laplace made some experiments to determine the elevation of water and of oil of oranges, and the depression of mercury in capillary tubes. Their results appear to have satisfied Laplace that the elevation or the depression of a fluid in capillary tubes varied inversely as the diameter of the tube. A tube of one millimetre in diameter gave a mean elevation of $13^{mm}\cdot569$ for water, and of $6^{mm}\cdot7389$ for oil of oranges, and a mean depression of $7^{mm}\cdot333$ for mercury.

In the *Supplément à la Théorie de l'Action Capillaire,* Laplace found the following expression for the approximate thickness (q) of a large drop of fluid resting on a horizontal plane[b]:

$$q + \frac{1}{ab} = \sqrt{\frac{2}{\alpha}} \sin\frac{\varpi'}{2} + \frac{1 - \cos^3\frac{\varpi'}{2}}{3\alpha l \sin\frac{\varpi'}{2}}.$$

For comparison, Gay-Lussac measured the thickness of a drop of mercury one decimetre ($2l$) in diameter resting upon a perfectly horizontal glass plane, and found it to be $3^{mm}\cdot378$ at a temperature $12^{\circ}\cdot8$ C. In calculating the value of q Laplace neglects the value of $\frac{1}{ab}$ because it is an insensible quantity. He then supposes $\frac{2}{\alpha} = 13$ square millimetres, and $\varpi' = 152$ grades $= 136^{\circ}\cdot8$ as determined by some *previous* experiments, and substituting finds $q = 3^{mm}\cdot39664$, instead of the measured thickness $3^{mm}\cdot378$.

Gauss merely refers to the results of Laplace, and gives the value of his α^2 which is equivalent to the $\frac{1}{2\alpha}$ of Laplace, equal 3·25 square millimetres.

[a] *Supp. au X Livre,* p. 52, 53.

[b] P. 64.

Poisson[a] obtains the following expression for the approximate theoretical thickness (k) of a drop of fluid resting on a horizontal plane:

$$k = a\sqrt{2}\cos\frac{\omega'}{2} - \frac{a^2}{\mu} + \frac{a^2}{3l'\cos\frac{\omega'}{2}}\left(1 - \sin^3\frac{\omega'}{2}\right) \ldots\ldots\ldots\ldots\ldots (o).$$

Here the a, and ω' of Poisson are respectively the $\sqrt{\frac{1}{\alpha}}$ and $\pi - \varpi'$ of Laplace. Referring to a previous experiment, Poisson writes $a^2\cos\omega' = 4{\cdot}5746$ for a temperature of $12^\circ{\cdot}8$ C., and for a first approximation he uses only the first term in (o). Thus

$$k^2 = \left(a\sqrt{2}\cos\frac{\omega'}{2}\right)^2 = a^2(1+\cos\omega'),$$

or

$$k^2\cos\omega' = (a^2\cos\omega')(1+\cos\omega').$$

And writing for k, $3^{\text{mm}}{\cdot}378$, the experimental thickness of a drop of mercury of radius $l = 50^{\text{mm}}$, at a temperature $12^\circ{\cdot}8$ C., as found by Gay-Lussac, he obtains

$$(3{\cdot}378)^2\cos\omega' = a^2\cos\omega'(1+\cos\omega') = 4{\cdot}5746\,(1+\cos\omega'),$$

which gives $\cos\omega' = \cos 48^\circ$ nearly, or $\omega' = 48^\circ$ nearly, and $a^2\cos\omega' = a^2\cos 48^\circ = 4{\cdot}5746$ now gives a or $\sqrt{\frac{1}{\alpha}} = 2^{\text{mm}}{\cdot}6146$.

In the next place the term $\frac{a^2}{\mu}$ only is neglected, because it is insensible:

$$l' = l + (\sqrt{2}-1)\,a = 50 + 1{\cdot}083 = 51{\cdot}083; \text{ and } k = 3^{\text{mm}}{\cdot}378.$$

Substituting in (o), ω' is found to be $45^\circ\ 30'$, which gives by the help of the equation $a^2\cos\omega' = 4{\cdot}5746$, a^2 or $\frac{1}{\alpha} = 6{\cdot}5262$ square millimetres, and a or $\sqrt{\frac{1}{\alpha}} = 2^{\text{mm}}{\cdot}5547$.

Avogadro[b] made numerous experiments to clear up some doubtful points relative to capillary action. He carefully examined how far any air or moisture commonly supposed to adhere to the interior of glass tubes might affect the depression of mercury. With this object in view, he exhausted the air, and heated the glass tube when the mercury was not in contact with it, and he found that the depression of the mercury in the tube was precisely the same after as it was before these precautions were taken.

In order however to determine the capillary constant a^2 or $\frac{1}{\alpha}$, for mercury, he made use of a tube of copper 20^{mm} long, and $2^{\text{mm}}{\cdot}80$ in diameter[c], well amalgamated

[a] *Nouvelle Théorie*, p. 217.

[b] *Accad. Fis. e Mat. Torino*, T. 40 (1836).

[c] *Ibid.* p. 221.

in the interior, and found it to be 5·56 square millimetres[a], and, therefore, a or $\sqrt{\frac{1}{\alpha}} = 2^{\mathrm{mm}}\cdot357$. Then substituting this value of a^2 in Poisson's[b] formula

$$h = -\frac{a^2 b}{\alpha} + \frac{\alpha}{b^3}\left[b^2 + \tfrac{2}{3}(1-b^2)^{\frac{3}{2}} - \tfrac{2}{3}\right],$$

and making $h = 4^{\mathrm{mm}}\cdot69$, α = radius of tube $= 0^{\mathrm{mm}}\cdot9525$, according to Gay-Lussac's experiment, he obtained $b = \cos\omega' = 0\cdot8440$ or $\omega' = 32^\circ\cdot5$ [c] $= (180^\circ - 147^\circ\cdot5)$ nearly, instead of $45^\circ\cdot5$ given by Poisson.

Substituting these two values $a^2 = 5\cdot56$ and $\omega' = 32^\circ\cdot5$ in Poisson's expression (o), for the theoretical thickness of a large drop of mercury quoted above, he obtains $3^{\mathrm{mm}}\cdot235$ instead of the measured thickness $3^{\mathrm{mm}}\cdot378$. Upon this he remarks that the smallness of this difference which corresponds to considerable differences in the values of a^2 and of $\cos\omega'$, shews that this observation was little adapted to give, by its combination with the depression of mercury in capillary tubes, exact values of these quantities.

Avogadro then determined to measure the depression of mercury in a capillary tube, so that he might obtain a value of ω' determined entirely from his own experiments. His glass tube had a radius of $0^{\mathrm{mm}}\cdot80$ [d]. He adopted a depression of $5^{\mathrm{mm}}\cdot125$, that being the mean of a great number of careful observations. The temperature was between 10° C. and 14° C. This depression is rather less than that found by Gay-Lussac quoted above, when allowance is made for difference in the radii of the tubes with which they experimented. Substituting as before he finds

$$\omega' = 40^\circ\,21' = (180^\circ - 139^\circ\,39').$$

In the next place Avogadro substitutes the value of $\cos\omega'$ just found $= 0\cdot7621$ and $a^2 = 5\cdot56$, in Poisson's formula (o) quoted above, and finds $3^{\mathrm{mm}}\cdot154$ for the thickness of a large drop of mercury instead of Gay-Lussac's measured thickness $3^{\mathrm{mm}}\cdot378$.

Desains[e] has deduced from Danger's experiments[f] a^2 or $\frac{1}{\alpha} = 6\cdot7144$, which gives a or $\sqrt{\frac{1}{\alpha}} = 2^{\mathrm{mm}}\cdot5912$ and $\omega' = 37^\circ\,52'\,33'' = (180^\circ - 142^\circ\,7'\,27'')$, which values appeared to satisfy best the whole of the experiments. He states however that for different sorts of mercury a or $\sqrt{\frac{1}{\alpha}}$ varied from $2^{\mathrm{mm}}\cdot55$ to $2^{\mathrm{mm}}\cdot61$, and ω' from 38° to 45° or from (180° − 142°) to (180° − 135°). Desains also obtained from experiments with large drops of mercury a or $\sqrt{\frac{1}{\alpha}} = 2^{\mathrm{mm}}\cdot621$ and $\omega' = 41^\circ\,36'\,30'' = (180^\circ - 138^\circ\,23'\,30'')$.

Still more recently Quincke has made very numerous experiments with a view to determine the capillary constants for a variety of fluids, and also for metals at

[a] P. 221.
[b] *Nouvelle Théorie*, p. 147.
[c] *Accad. Fis. e Mat.* p. 223.
[d] P. 227.
[e] *Ann. de Ch. Ph.* [3] T. LI. (1857).
[f] *Ann. de Ch. Ph.* [3] T. XXIV. p. 501.

a temperature just above the melting point. He found that the values of a or $\sqrt{\frac{1}{\alpha}}$ decreased for the same drop of mercury[a], according to the time it had stood in position. He also found that ω' varied from 38° to 45°[b], or from $(180^\circ - 142^\circ)$ to $(180^\circ - 135^\circ)$. But other results were obtained far beyond these limits. For the mean value of a or $\sqrt{\frac{1}{\alpha}}$ he adopted $2^{mm}\cdot 8$[c], and some of his experiments gave as high a value as $2^{mm}\cdot 9$, both of which differ considerably from the previously received value $2^{mm}\cdot 6$.

In 1868-9 Quincke published[d] the results of some experiments made to determine the capillary constants at the common surfaces of two fluids incapable of mixing. In this case he pursued methods of experimenting in some respects similar to those I had suggested in my application to the Royal Society in 1855. But the value of Quincke's results is very much diminished by the manner in which he carried out his experiments, and by his mode of determining the theoretical forms of sessile drops of fluid. Thus Quincke's method requires the measurement, with great precision, of the height of the vertex of a large drop above the largest horizontal section of the drop. But in my experiments I have found that only a rough approximation to this quantity can be obtained directly by the most careful measurement. The theoretical forms of Quincke are much the same as those of Segner, for in the calculations of both, one of the two principal radii of curvature is supposed to be infinite. There is also a further objection to the use of large drops of fluid, which Quincke's methods of calculation necessitated, because they change their form slowly when a change in their volume is made. But only a slight change in the volume of a small drop will give a marked change in its form.

The favourite method of testing the theories of capillary action has been by the measurements of the heights to which fluids *rise* in capillary tubes. In cases where the fluid wets the solid, there is only one constant, α, to be determined, as the angle $\omega' = 0$. But experiments of this kind are very liable to be vitiated by irregularities in the bore of the tubes, or by impurities adhering to the inner surface of fine tubes, which do not admit of being cleaned. The layer of fluid which lines the tubes must make a sensible reduction in the radii of the finer capillary tubes. And the theoretical expressions for the height of the fluids in these tubes are *approximations* which are not strictly applicable to tubes of large diameter used in experiments of this kind.

Some recent writers on capillary action have disputed the correctness of the results arrived at by the earlier experimenters. Thus Simon[e] has concluded from numerous experiments of his own that the elevation of water in capillary tubes is very far from varying inversely as their diameters, and that the height to which water rises between parallel plates compared with that which takes place in tubes, instead of being as 1 : 2, is as 1 : 3, or rather as 1 to π.

[a] *Pogg. Ann.* Bd. cv., p. 35 (1858).
[b] P. 45.
[c] P. 47.
[d] *Pogg. Ann.* Bd. cxxxix.
[e] *Ann. de Ch. Ph.* [3] T. xxxviii. (1851).

Bède[a] comes to the conclusion that the depression of mercury and the elevation of water in glass tubes do not respectively vary inversely as the diameters of the tubes exactly, and that the thickness of the substance of the tubes has a sensible effect, or, in other words, that the molecular attractions are not insensible at sensible distances.

Wolf[b] afterwards concluded from his experiments that the elevation of the same fluid in capillary tubes, all circumstances being alike in other respects, depends upon the nature of the tube.

Laplace and Poisson considered that the only effect of a change of temperature was to change the elevation of a capillary column according to the change in density. Thus Laplace[c] says "L'élévation d'un fluide qui mouille exactement les parois d'un "tube capillaire, est, à diverses températures, en raison directe de la densité du "fluide, et en raison inverse du diamètre intérieur du tube." And Poisson[d] obtains for the elevation (h) of a fluid in a capillary tube of radius α

$$h = \frac{\pi}{4g\rho\alpha}\int_0^\infty Rr^4 dr.$$

He then supposes that by a change of temperature h, ρ and R are respectively changed into h', ρ' and R', neglecting the change in α. And having found $\frac{h'}{h} = \frac{\rho'}{\rho}$ he remarks "L'expérience montre, en effet, que pour un même liquide à différentes "températures, l'élévation du point C croît proportionellement à la densité; ce qui "donne lieu de croire que la force répulsive de la chaleur, ou du moins, sa "variation, que nous avons négligée, n'a qu'une influence insensible sur l'intégrale "$\int_0^\infty Rr^4 dr$."

Very careful experiments have been carried out by Frankenheim and Sondhauss, and afterwards by Brunner, to determine how far the height of the capillary column depends upon the temperature. Frankenheim[e] found that the height to which water rises in a capillary tube $1^{mm}\cdot 0$ in radius at a temperature t° C. is

$$15^{mm}\cdot 336 - 0\cdot 02751t - 0\cdot 000014t^2 \text{ between } -2^\circ\cdot 5 \text{ and } 93^\circ\cdot 4\text{ C.},$$

and Brunner[f] finds it to be

$$15^{mm}\cdot 33215 - 0\cdot 0286396t \text{ from } 0^\circ \text{ to } 82^\circ\text{ C.}$$

Hence it appears that the elevation of fluids decreases with an increase of temperature much more rapidly than would be expected according to the theories of Laplace and Poisson.

In the foregoing sketch of the progress of experiments made to determine capillary constants I have given attention chiefly to those where mercury was

[a] *Savans Etr. Brux.* T. xxv. (1853).
[b] *Ann. de Ch. Ph.* [3] T. xlix.
[c] *Supp. Th. de l'Action Capillaire*, p. 39.
[d] *Nouvelle Théorie*, p. 106.
[e] *Pogg. Ann.* Bd. lxxii. (1847).
[f] *Disquisitio Phys. Exp.*, p. 34, 35 (1846).

the fluid employed. Every experimenter finds that changes of form are constantly going on in capillary surfaces from one cause or another. Still something more definite is desirable in the results. But as the experiments have been conducted apparently with every precaution, it does not appear probable that any new experiments of the same kind would lead to better results. When ω' is determined by reflection its value must be obtained for a point at a short distance from the junction of the solid and fluid surfaces. The experiments on the thicknesses of large drops of fluid are not satisfactory because the theoretical expression is not exact, and because the thickness of the drop varies so slowly in large drops. Also the approximate theoretical thickness is given in terms of two unknown quantities a and ω'.

During the time when I was able to use the Cambridge University Library, I made copious extracts from numerous papers on this subject, but it does not appear necessary for me to allude further to them in this place, especially as the late Professor Challis has published a very good and elaborate report on Capillary Action[a]. For numerous references to the works of early writers on the subject, reference may be made to the articles "Capillarität," "Cohäsion" and "Tropfen" in Gehler's *Physikalisches Wörterbuch*. Recent experiments will be found referred to in *Fortschritte der Physik* 1845, &c. and in *Jahresbericht*, 1847, &c. von Liebig, Kopp, u. Will. See also the article on Capillary Action in the 9th edition of the *Encyclopædia Britannica* by the late Professor Clerk Maxwell.

[a] *Brit. Ass. Report*, 1834.

CHAPTER III.

ON THE CALCULATION OF THE THEORETICAL FORMS OF DROPS OF FLUID, UNDER THE INFLUENCE OF CAPILLARY ACTION, WHEN SUCH DROPS ARE BOUNDED BY SURFACES OF REVOLUTION WHICH MEET THEIR RESPECTIVE AXES AT RIGHT ANGLES.

WE have already stated that various methods of obtaining the differential equation to the surface of fluid under the action of capillary forces have been given by Laplace and other writers on Capillary Action. The form of the equation obtained by these different methods is, however, in all cases the same.

Perhaps the simplest way of obtaining the equation in question is to consider the fluid to be in equilibrium under the action of gravity and of a uniform surface tension.

Let T be this uniform tension, R and R' the principal radii of curvature at any point of the surface of the fluid, p the fluid pressure at that point.

Then
$$\frac{1}{R}+\frac{1}{R'}=\frac{p}{T}.$$

If z be the vertical coordinate of the point measured downwards, σ the density of the fluid, and g the force of gravity, then

$$p = g\sigma z + C, \text{ where } C \text{ is a constant.}$$

When two different fluids are separated by the capillary surface, p is the difference of the pressures in the two fluids at their point of meeting, and σ is the difference of the densities of the fluids.

When a drop rests upon or hangs from a horizontal plane surface, the remaining surface of the drop being free, this free surface will evidently be one of revolution about a vertical axis, and it will meet the axis at right angles.

Take the axis of revolution as the axis of z, and the point in which it meets the free surface as the origin.

Let x be the horizontal and z the vertical coordinate of any point in a meridional section of the surface of the fluid, ρ the radius of curvature of the meridional section at that point, and ϕ the angle which the normal to the surface makes with the axis of revolution.

Then the length of the normal terminated by the axis is $\dfrac{x}{\sin\phi}$, and we have

$$R = \rho,\ R' = \frac{x}{\sin\phi},$$

and the above found equation becomes

$$\frac{1}{\rho} + \frac{\sin\phi}{x} = \frac{C + g\sigma z}{T}.$$

Let b be the radius of curvature at the origin, so that at that point we have both

$$\rho = b, \text{ and limit } \left(\frac{x}{\sin\phi}\right) = b.$$

Hence
$$\frac{C}{T} = \frac{2}{b}$$

and the equation becomes

$$\frac{1}{\rho} + \frac{\sin\phi}{x} = \frac{2}{b} + \frac{g\sigma}{T} z,$$

or
$$\frac{b}{\rho} + \frac{\sin\phi}{\left(\frac{x}{b}\right)} = 2 + \frac{g\sigma b^2}{T}\left(\frac{z}{b}\right).$$

Let $\dfrac{g\sigma b^2}{T}$ be called β, which is an abstract number. Also let s be the length of the arc of the meridional section, measured from the origin and terminated at the point under consideration.

Then
$$ds = \rho d\phi,$$
$$dx = \rho\cos\phi d\phi,$$
$$dz = \rho\sin\phi d\phi;$$

or
$$d\left(\frac{s}{b}\right) = \left(\frac{\rho}{b}\right) d\phi,$$
$$d\left(\frac{x}{b}\right) = \left(\frac{\rho}{b}\right)\cos\phi d\phi,$$
$$d\left(\frac{z}{b}\right) = \left(\frac{\rho}{b}\right)\sin\phi d\phi.$$

For the sake of simplicity, we will write x, z, ρ and s instead of $\dfrac{x}{b}$, $\dfrac{z}{b}$, $\dfrac{\rho}{b}$ and $\dfrac{s}{b}$,

which amounts to taking the quantity b as the unit of length, and we may at any time re-introduce the quantity b by writing

$$\frac{x}{b},\ \frac{z}{b},\ \frac{\rho}{b} \text{ and } \frac{s}{b} \text{ instead of } x,\ z,\ \rho \text{ and } s.$$

Thus simplified, our equation becomes

$$\frac{1}{\rho}+\frac{\sin\phi}{x}=2+\beta z.$$

Also when $\phi=0$, we have $z=0$, $\rho=1$ and limit $\left(\frac{x}{\sin\phi}\right)=1$, hence the form of the curve depends on the single parameter β. The magnitude of the curve, or its *scale*, is proportional to b.

The same equation is applicable to the case of hanging drops, but in that case z is to be measured upwards from the vertex, and β will be negative.

Since
$$\frac{1}{\rho}=\frac{d^2z}{dx^2}\div\left\{1+\left(\frac{dz}{dx}\right)^2\right\}^{\frac{3}{2}}$$

and
$$\sin\phi=\frac{\dfrac{dz}{dx}}{\left\{1+\left(\dfrac{dz}{dx}\right)^2\right\}^{\frac{1}{2}}},$$

the above equation is equivalent to

$$\frac{d^2z}{dx^2}+\left\{1+\left(\frac{dz}{dx}\right)^2\right\}\frac{dz}{xdx}=(2+\beta z)\left\{1+\left(\frac{dz}{dx}\right)^2\right\}^{\frac{3}{2}},$$

a differential equation of the 2nd order. The two arbitrary constants which enter into the integral of this equation are to be determined by the condition that when $x=0$,

$$z=0, \quad \text{and } \frac{dz}{xdx}=1.$$

We are unable either to find the general relation between x and z, by means of this equation, or to express these two quantities in terms of a third variable.

We may, however, as in all cases where the differential equation to a curve is given, develope the increments of the coordinates in series proceeding according to ascending powers of the increment of the quantity chosen as the independent variable. Thus we can trace a small portion of the curve starting from a known point, and then we may make the point which terminates this portion a new starting point for tracing another small portion, and so on successively until any required portion of the curve has been traced.

For instance, suppose the given equation to be

$$\frac{d^2y}{dt^2}=f\left(\frac{dy}{dt},\ y,\ t\right),$$

where f denotes any function of the quantities $\frac{dy}{dt}$, y and t.

Then by repeated differentiations of this equation, and by substitution of the value of $\frac{d^2y}{dt^2}$ in the successive results, we may find the general values of the higher differential coefficients

$$\frac{d^3y}{dt^3},\ \frac{d^4y}{dt^4},\ \&c.$$

in terms of $\frac{dy}{dt}$, y and t.

Hence if, for a given value t_0 of t, we know that

$$y = y_0 \text{ and } \frac{dy}{dt} = \left(\frac{dy}{dt}\right)_0, \text{ suppose,}$$

we can find the values of $\frac{d^2y}{dt^2}$ and the higher differential coefficients of y, which correspond to $t = t_0$.

Let these values be denoted by $\left(\frac{d^2y}{dt^2}\right)_0$, $\left(\frac{d^3y}{dt^3}\right)_0$, &c.

Therefore if $t_1 = t_0 + \delta t_0$, and if y_1 and $\left(\frac{dy}{dt}\right)_1$ be the values of y and $\frac{dy}{dt}$ which correspond to $t = t_1$, we have by Taylor's theorem

$$y_1 = y_0 + \left(\frac{dy}{dt}\right)_0 \delta t_0 + \frac{1}{1\,.\,2}\left(\frac{d^2y}{dt^2}\right)_0 \delta t_0^{\,2} + \frac{1}{1\,.\,2\,.\,3}\left(\frac{d^3y}{dt^3}\right)_0 \delta t_0^{\,3} + \&c.,$$

and $$\left(\frac{dy}{dt}\right)_1 = \left(\frac{dy}{dt}\right)_0 + \left(\frac{d^2y}{dt^2}\right)_0 \delta t_0 + \frac{1}{1\,.\,2}\left(\frac{d^3y}{dt^3}\right)_0 \delta t_0^{\,2} + \frac{1}{1\,.\,2\,.\,3}\left(\frac{d^4y}{dt^4}\right)_0 \delta t_0^{\,3} + \&c.$$

The increment δt_0 must be taken so small as to render these series convergent.

The values of y_1 and $\left(\frac{dy}{dt}\right)_1$ being thus known, we may find $\left(\frac{d^2y}{dt^2}\right)_1$, $\left(\frac{d^3y}{dt^3}\right)_1$, &c., by the same formulæ as before; and then if

$$t_2 = t_1 + \delta t_1,$$

and if y_2 and $\left(\frac{dy}{dt}\right)_2$ be the values of y and $\frac{dy}{dt}$ which correspond to $t = t_2$, we may similarly find y_2 and $\left(\frac{dy}{dt}\right)_2$, and the same process may be repeated as often as we please.

A similar process may be employed if we have any number of simultaneous differential equations, and the same number of dependent variables, such as, for instance, the following:

$$\frac{dx}{dt} = f(x, y, t),$$

$$\frac{dy}{dt} = F(x, y, t).$$

The method fails if any of the differential coefficients employed become infinite in the interval over which the integrations extend, and therefore the independent variable should be so chosen that no infinite or very large values of the differential coefficients will be introduced.

The intervals adopted should be so small that a few of the terms of the series will suffice to give the results with all the accuracy that is desired.

After a few points of the curve, in the neighbourhood of the starting point, have been determined by the foregoing or some equivalent method, it will usually be found more convenient to determine other points of the curve in succession by making use of a series of successive values of the differential coefficient which is given immediately by the differential equation, rather than by employing the values of the successive differential coefficients of higher orders which are found by means of the several derived equations.

To fix the ideas we will suppose, with especial reference to our present problem, that the given differential equation is one of the first order, say

$$\frac{dy}{dt} = q = f(y,\ t).$$

Let $\ldots\ t_{-4},\ t_{-3},\ t_{-2},\ t_{-1},\ t_0,\ t_1$, &c. be a series of values of the independent variable t, forming an arithmetical progression with the common difference ω.

Let $$\ldots\ y_{-4},\ y_{-3},\ y_{-2},\ y_{-1},\ y_0,\ y_1, \text{ \&c.}$$
denote the corresponding values of y, and let
$$\ldots\ q_{-4},\ q_{-3},\ q_{-2},\ q_{-1},\ q_0,\ q_1, \text{ \&c.}$$
be the corresponding values of q, or of $\dfrac{dy}{dt}$,

and suppose ω to be so small that the successive differences of these values of q soon become small enough to be neglected.

Let $$t = t_0 + n\omega,$$
and suppose that we have already found the values of
$$\ldots\ y_{-4},\ y_{-3},\ y_{-2},\ y_{-1} \text{ up to } y_0$$
and therefore also those of $\ldots\ q_{-4},\ q_{-3},\ q_{-2},\ q_{-1}$ up to q_0,

and that the successive differences of these quantities are taken according to the following scheme:

$$\begin{array}{ccccccc}
n & q & & & & & \\
\ldots & \ldots & & \ldots & & & \\
 & & \ldots & & \ldots & & \\
-4 & q_{-4} & & \ldots & & \ldots & \\
 & & \Delta q_{-3} & & \ldots & & \\
-3 & q_{-3} & & \Delta^2 q_{-2} & & \ldots & \text{\&c.} \\
 & & \Delta q_{-2} & & \Delta^3 q_{-1} & & \\
-2 & q_{-2} & & \Delta^2 q_{-1} & & \Delta^4 q_0 & \text{\&c.} \\
 & & \Delta q_{-1} & & \Delta^3 q_0 & & \\
-1 & q_{-1} & & \Delta^2 q_0 & & & \\
 & & \Delta q_0 & & & & \\
0 & q_0 & & & & &
\end{array}$$

Then the general value of q found by the ordinary formula of interpolation, for any value of n, will be

$$q = q_0 + \Delta q_0 \frac{n}{1} + \Delta^2 q_0 \frac{n(n+1)}{1.2} + \Delta^3 q_0 \frac{n(n+1)(n+2)}{1.2.3} + \Delta^4 q_0 \frac{n(n+1)(n+2)(n+3)}{1.2.3.4} + \&c.$$

provided that n be taken between limits for which this series remains convergent.

Hence the general value of y will be

$$y = \int q dt = \omega \int q dn,$$

or, substituting the above value of q, and adding a constant to the integral so as to make $y = y_0$ when $n = 0$,

$$y = y_0 + \omega \left\{ q_0 n + \Delta q_0 \frac{n^2}{2} + \Delta^2 q_0 \int \frac{n(n+1)}{1.2} dn + \Delta^3 q_0 \int \frac{n(n+1)(n+2)}{1.2.3} dn + \&c. \right\},$$

where all the integrals are supposed to vanish when $n = 0$.

If, in particular, we put $n = -1$, and substitute the several values of the definite integrals

$$\int_0^{-1} \frac{n(n+1)}{1.2} dn, \quad \int_0^{-1} \frac{n(n+1)(n+2)}{1.2.3} dn, \ \&c.$$

we shall have, by changing the signs throughout,

$$y_0 - y_{-1} = \omega \left\{ q_0 - \frac{1}{2} \Delta q_0 - \frac{1}{12} \Delta^2 q_0 - \frac{1}{24} \Delta^3 q_0 - \frac{19}{720} \Delta^4 q_0 - \frac{3}{160} \Delta^5 q_0 - \frac{863}{60480} \Delta^6 q_0 \right.$$
$$\left. - \frac{275}{24192} \Delta^7 q_0 - \frac{33953}{3628800} \Delta^8 q_0 - \frac{8183}{1036800} \Delta^9 q_0 - \&c. \right\}.$$

Similarly, putting $n = 1$ and substituting the values of the definite integrals

$$\int_0^1 \frac{n(n+1)}{1.2} dn, \quad \int_0^1 \frac{n(n+1)(n+2)}{1.2.3} dn, \ \&c.$$

we shall have

$$y_1 - y_0 = \omega \left\{ q_0 + \frac{1}{2} \Delta q_0 + \frac{5}{12} \Delta^2 q_0 + \frac{3}{8} \Delta^3 q_0 + \frac{251}{720} \Delta^4 q_0 + \frac{95}{288} \Delta^5 q_0 + \frac{19087}{60480} \Delta^6 q_0 \right.$$
$$\left. + \frac{5257}{17280} \Delta^7 q_0 + \frac{1070017}{3628800} \Delta^8 q_0 + \frac{2082753}{7257600} \Delta^9 q_0 + \&c. \right\}.$$

It will usually be found expedient to choose ω so small as to render it unnecessary to proceed beyond the fourth order of differences.

The series last found gives the value of y_1 in terms of quantities which are all supposed to be already known, that is, the value of the variable y which was previously known for values of the independent variable extending as far as $t = t_0$, now becomes known for the value $t = t_0 + \omega$, or at the end of an additional interval ω.

It will be remarked, however, that the coefficients of the series above found for $y_0 - y_{-1}$, after the first two terms, are much smaller and diminish much more rapidly than the corresponding coefficients of the series for $y_1 - y_0$. Hence by taking into account the same number of terms of the series in the two cases, the value of $y_0 - y_{-1}$ will be determined with much greater accuracy than that of $y_1 - y_0$.

In what has gone before, the successive values of y up to y_0 are supposed to be already known, and therefore the equation which gives the value of $y_0 - y_{-1}$ may be regarded as merely supplying a verification of former work. If, however, we suppose that the value of y_0 is only approximately known, while the successive values as far as y_{-1} have been found with the degree of accuracy desired, we may use the equation for $y_0 - y_{-1}$ to give the corrected value of y_0, in the following manner.

Suppose that (y_0) is an approximate value of y_0, and let $y_0 = (y_0) + \eta$, where η is so small that its square may be neglected.

Also let (q_0) be the corresponding approximate value of q_0 found from the equation

$$q = f(y, t)$$

by putting $y = (y_0)$ and $t = t_0$.

Then we may put
$$q_0 = (q_0) + k\eta,$$

where k denotes the value of the partial differential coefficient $\dfrac{dq}{dy}$ or $\dfrac{df(y, t)}{dy}$ found by substituting (y_0) for y and t_0 for t after the differentiation.

Let $\Delta(q_0)$, $\Delta^2(q_0)$, $\Delta^3(q_0)$, $\Delta^4(q_0)$, &c. denote the values of the successive differences formed with the approximate value (q_0) and the known values q_{-1}, q_{-2}, &c. which immediately precede it, then we have

$$\begin{aligned}\Delta q_0 &= \Delta(q_0) + k\eta,\\ \Delta^2 q_0 &= \Delta^2(q_0) + k\eta,\\ \Delta^3 q_0 &= \Delta^3(q^0) + k\eta,\\ \&c. &= \&c.\end{aligned}$$

But, by the equation before obtained,

$$y_0 - y_{-1} = \omega\left\{q_0 - \frac{1}{2}\Delta q_0 - \frac{1}{12}\Delta^2 q_0 - \frac{1}{24}\Delta^3 q_0 - \frac{19}{720}\Delta^4 q_0 - \frac{3}{160}\Delta^5 q_0 - \frac{863}{60480}\Delta^6 q_0\right.$$
$$\left. - \frac{275}{24192}\Delta^7 q_0 - \frac{33953}{3628800}\Delta^8 q_0 - \frac{8183}{1036800}\Delta^9 q_0 - \&c.\right\}.$$

Or, substituting for y_0, q_0, Δq_0, $\Delta^2 q_0$, &c. their values in terms of η and known quantities,

$$(y_0) - y_{-1} + \eta = \omega\left\{(q_0) - \frac{1}{2}\Delta(q_0) - \frac{1}{12}\Delta^2(q_0) - \frac{1}{24}\Delta^3(q_0) - \frac{19}{720}\Delta^4(q_0) - \&c.\right\}$$
$$+ \omega k\eta\left\{1 - \frac{1}{2} - \frac{1}{12} - \frac{1}{24} - \frac{19}{720} - \&c.\right\}.$$

Hence if ϵ denote the excess of the quantity

$$\omega\left\{(q_0) - \frac{1}{2}\Delta(q_0) - \frac{1}{12}\Delta^2(q_0) - \frac{1}{24}\Delta^3(q_0) - \frac{19}{720}\Delta^4(q_0) - \&c.\right\}$$

over the quantity $(y_0) - y_{-1}$, we shall have

$$\eta = \epsilon + \omega k\eta\left\{1 - \frac{1}{2} - \frac{1}{12} - \frac{1}{24} - \frac{19}{720} - \&c.\right\}$$

or

$$\eta = \frac{\epsilon}{1 - \omega k\left[1 - \frac{1}{2} - \frac{1}{12} - \frac{1}{24} - \frac{19}{720} - \&c.\right]},$$

which determines η, and therefore $y_0 = (y_0) + \eta$, and $q_0 = (q_0) + k\eta$ both become known.

If in finding ϵ we stop at the term involving $\Delta^4(q_0)$, we shall have

$$\eta = \frac{\epsilon}{1 - \frac{251}{720}\omega k},$$

and

$$k\eta = \frac{k\epsilon}{1 - \frac{251}{720}\omega k}.$$

It will be observed that the coefficient of ωk in the denominator of these expressions is the same as that of $\omega\Delta^4 q_0$ in the expression for $y_1 - y_0$.

This is no mere coincidence, as it is easy to shew that, generally, the coefficient of any term

$$\omega\Delta^r q_0,$$

in the expression for $y_1 - y_0$, is equal to the sum of the coefficients of the terms involving

$$\omega q_0,\ \omega\Delta q_0,\ \omega\Delta^2 q_0,\ \&c. \ldots \omega\Delta^r q_0$$

in the expression for $y_0 - y_{-1}$.

Hence if in finding ϵ we also include the term involving $\Delta^5(q_0)$, we shall similarly have

$$\eta = \frac{\epsilon}{1 - \frac{95}{288}\omega k},$$

and

$$k\eta = \frac{k\epsilon}{1 - \frac{95}{288}\omega k}.$$

An approximate value of y_0 may always be found from the series of values $\ldots y_{-4}, y_{-3}, y_{-2}, y_{-1}$ previously calculated, by taking the successive differences of four or five of the last terms of the series, and assuming that the last difference so found remains constant.

The numerical operations will be greatly facilitated by the use of Tables which exhibit the values of

$$\frac{19}{720}\Delta^4 q,\quad \frac{3}{160}\Delta^5 q,\quad \frac{863}{60480}\Delta^6 q, \text{ \&c.}$$

for given values of $\Delta^4 q,\ \Delta^5 q,\ \Delta^6 q$, &c.

Such Tables have been formed by Mr Bashforth for this purpose, and are given at the end of this Chapter.

Having made these preliminary observations on the general method of finding successive small portions of a curve by means of its differential equation, we will now proceed to apply the method to the problem under consideration, viz. to the tracing of the curve formed by a meridional section of a drop of fluid, by means of the equation above found

$$\frac{1}{\rho}+\frac{\sin\phi}{x}=2+\beta z.$$

First, suppose ϕ to be taken as the independent variable.

The above equation may be regarded as giving ρ as a function of the coordinates x and z, and these latter quantities are to be found by the integration of the equations

$$\frac{dx}{d\phi}=\rho\cos\phi,$$

$$\frac{dz}{d\phi}=\rho\sin\phi.$$

Also x and z vanish with ϕ, and ρ is initially $=1$.

We will first find the form of the curve in the neighbourhood of the origin by developing ρ and the coordinates x and z in series of ascending powers of ϕ.

Instead of employing the general method described at the outset, it will be found more convenient, in this particular case, to proceed as follows:

Assume, as we evidently may do,

$$\rho=1+b_2\phi^2+b_4\phi^4+b_6\phi^6+b_8\phi^8+b_{10}\phi^{10}+\text{\&c.},$$

where b_2, b_4, &c. are constants to be determined, then

$$\frac{dx}{d\phi}=\rho\cos\phi=\rho\left\{1-\frac{1}{1.2}\phi^2+\frac{1}{1.2.3.4}\phi^4-\frac{1}{1.2\ldots6}\phi^6\right.$$
$$\left.+\frac{1}{1.2\ldots8}\phi^8-\frac{1}{1.2\ldots10}\phi^{10}+\text{\&c.}\right\}.$$

Substitute the assumed value of ρ and integrate, therefore

$$x = \left[\phi - \frac{1}{6}\phi^3 + \frac{1}{120}\phi^5 - \frac{1}{5040}\phi^7 + \frac{1}{362880}\phi^9 - \frac{1}{39916800}\phi^{11} + \&c.\right]$$

$$+ b_2\left[\frac{1}{3}\phi^3 - \frac{1}{10}\phi^5 + \frac{1}{168}\phi^7 - \frac{1}{6480}\phi^9 + \frac{1}{443520}\phi^{11} - \&c.\right]$$

$$+ b_4\left[\frac{1}{5}\phi^5 - \frac{1}{14}\phi^7 + \frac{1}{216}\phi^9 - \frac{1}{7920}\phi^{11} + \&c.\right]$$

$$+ b_6\left[\frac{1}{7}\phi^7 - \frac{1}{18}\phi^9 + \frac{1}{264}\phi^{11} - \&c.\right]$$

$$+ b_8\left[\frac{1}{9}\phi^9 - \frac{1}{22}\phi^{11} + \&c.\right]$$

$$+ b_{10}\left[\frac{1}{11}\phi^{11} - \&c.\right]$$

$$+ \&c., \quad \&c.$$

Similarly

$$\frac{dz}{d\phi} = \rho\left\{\phi - \frac{1}{6}\phi^3 + \frac{1}{120}\phi^5 - \frac{1}{5040}\phi^7 + \frac{1}{362880}\phi^9 - \frac{1}{39916800}\phi^{11} + \&c.\right\},$$

and therefore

$$z = \left[\frac{1}{2}\phi^2 - \frac{1}{24}\phi^4 + \frac{1}{720}\phi^6 - \frac{1}{40320}\phi^8 + \frac{1}{3628800}\phi^{10} - \frac{1}{479001600}\phi^{12} + \&c.\right]$$

$$+ b_2\left[\frac{1}{4}\phi^4 - \frac{1}{36}\phi^6 + \frac{1}{960}\phi^8 - \frac{1}{50400}\phi^{10} + \frac{1}{4354560}\phi^{12} - \&c.\right]$$

$$+ b_4\left[\frac{1}{6}\phi^6 - \frac{1}{48}\phi^8 + \frac{1}{1200}\phi^{10} - \frac{1}{60480}\phi^{12} + \&c.\right]$$

$$+ b_6\left[\frac{1}{8}\phi^8 - \frac{1}{60}\phi^{10} + \frac{1}{1440}\phi^{12} - \&c.\right]$$

$$+ b_8\left[\frac{1}{10}\phi^{10} - \frac{1}{72}\phi^{12} + \&c.\right]$$

$$+ b_{10}\left[\frac{1}{12}\phi^{12} - \&c.\right].$$

$$+ \&c., \quad \&c.$$

Also, we find

$$\frac{1}{\rho} = 1 - b_2\phi^2 + (b_2^2 - b_4)\phi^4 - (b_2^3 - 2b_2b_4 + b_6)\phi^6 + (b_2^4 - 3b_2^2b_4 + b_4^2 + 2b_2b_6 - b_8)\phi^8$$
$$- (b_2^5 - 4b_2^3b_4 + 3b_2b_4^2 + 3b_2^2b_6 - 2b_4b_6 - 2b_2b_8 + b_{10})\phi^{10} + \&c.$$

Also
$$\frac{\sin\phi}{x} = \left(\frac{\sin\phi}{\phi}\right) \div \left(\frac{x}{\phi}\right),$$

and
$$\frac{\sin\phi}{\phi} = 1 - \frac{1}{6}\phi^2 + \frac{1}{120}\phi^4 - \frac{1}{5040}\phi^6 + \frac{1}{362880}\phi^8 - \frac{1}{39916800}\phi^{10} + \&c.,$$

and from above

$$\begin{aligned}\frac{x}{\phi} = 1 &- \left(\frac{1}{6} - \frac{1}{3}b_2\right)\phi^2 + \left(\frac{1}{120} - \frac{1}{10}b_2 + \frac{1}{5}b_4\right)\phi^4 - \left(\frac{1}{5040} - \frac{1}{168}b_2 + \frac{1}{14}b_4 - \frac{1}{7}b_6\right)\phi^6 \\ &+ \left(\frac{1}{362880} - \frac{1}{6480}b_2 + \frac{1}{216}b_4 - \frac{1}{18}b_8 + \frac{1}{9}b_8\right)\phi^8 \\ &- \left(\frac{1}{39916800} - \frac{1}{443520}b_2 + \frac{1}{7920}b_4 - \frac{1}{264}b_6 + \frac{1}{22}b_8 - \frac{1}{11}b_{10}\right)\phi^{10} \\ &+ \&c., \&c.\end{aligned}$$

Hence, by performing the division indicated, we may find

$$\begin{aligned}\frac{\sin\phi}{x} = 1 &- \frac{1}{3}b_2\phi^2 + \left(\frac{2}{45}b_2 + \frac{1}{9}b_2^2 - \frac{1}{5}b_4\right)\phi^4 \\ &+ \left(\frac{4}{945}b_2 - \frac{4}{135}b_2^2 - \frac{1}{27}b_2^3 + \frac{4}{105}b_4 + \frac{2}{15}b_2b_4 - \frac{1}{7}b_6\right)\phi^6 \\ &+ \left(\frac{2}{4725}b_2 - \frac{4}{4725}b_2^2 + \frac{2}{135}b_2^3 + \frac{1}{81}b_2^4 + \frac{16}{4725}b_4 - \frac{68}{1575}b_2b_4\right. \\ &\qquad \left. - \frac{1}{15}b_2^2b_4 + \frac{1}{25}b_4^2 + \frac{2}{63}b_8 + \frac{2}{21}b_2b_6 - \frac{1}{9}b_8\right)\phi^8 \\ &+ \left(\frac{4}{93555}b_2 + \frac{4}{42525}b_2^2 - \frac{8}{14175}b_2^3 - \frac{8}{1215}b_2^4 - \frac{1}{243}b_2^5\right. \\ &\qquad + \frac{52}{155925}b_4 - \frac{8}{14175}b_2b_4 + \frac{16}{525}b_2^2b_4 + \frac{4}{135}b_2^3b_4 - \frac{8}{525}b_4^2 \\ &\qquad - \frac{1}{25}b_2b_4^2 + \frac{4}{1485}b_6 - \frac{32}{945}b_2b_6 - \frac{1}{21}b_2^2b_6 + \frac{2}{35}b_4b_6 + \frac{8}{297}b_8 \\ &\qquad \left. + \frac{2}{27}b_2b_8 - \frac{1}{11}b_{10}\right)\phi^{10} \\ &+ \&c., \&c.\end{aligned}$$

Substitute these expressions in the equation
$$\frac{1}{\rho} + \frac{\sin\phi}{x} = 2 + \beta z,$$

and equate the coefficients of corresponding powers of ϕ, and we shall find successively

$$b_2 = -\frac{3}{8}\beta,$$

$$b_4 = \frac{1}{48}\beta + \frac{5}{24}\beta^2,$$

$$b_6 = -\frac{11}{5760}\beta - \frac{3}{128}\beta^2 - \frac{1183}{9216}\beta^3,$$

$$b_8 = -\frac{1}{8960}\beta + \frac{53}{18432}\beta^2 + \frac{2011}{92160}\beta^3 + \frac{6799}{81920}\beta^4,$$

$$b_{10} = -\frac{233}{14515200}\beta + \frac{1}{36288}\beta^2 - \frac{1469}{442368}\beta^3 - \frac{104513}{5529600}\beta^4 - \frac{4882031}{88473600}\beta^5,$$

which gives the value of ρ in terms of ϕ, as far as ϕ^{10}.

Again, substituting these values of b_2, b_4, &c., in the expressions for $\frac{1}{\rho}$, x and z, we shall obtain

$$\frac{1}{\rho} = 1 + \frac{3}{8}\beta\phi^2 + \left(-\frac{1}{48}\beta - \frac{13}{192}\beta^2\right)\phi^4 + \left(\frac{11}{5760}\beta + \frac{1}{128}\beta^2 + \frac{229}{9216}\beta^3\right)\phi^6$$
$$+ \left(\frac{1}{8960}\beta - \frac{31}{30720}\beta^2 - \frac{401}{92160}\beta^3 - \frac{8431}{737280}\beta^4\right)\phi^8$$
$$+ \left(\frac{233}{14515200}\beta - \frac{17}{725760}\beta^2 + \frac{1517}{2211840}\beta^3 + \frac{7409}{2764800}\beta^4 + \frac{522091}{88473600}\beta^5\right)\phi^{10}$$

to the 10th order in ϕ;

$$x = \phi - \left(\frac{1}{6} + \frac{1}{8}\beta\right)\phi^3 + \left(\frac{1}{120} + \frac{1}{24}\beta + \frac{1}{24}\beta^2\right)\phi^5 - \left(\frac{1}{5040} + \frac{23}{5760}\beta + \frac{7}{384}\beta^2 + \frac{169}{9216}\beta^3\right)\phi^7$$
$$+ \left(\frac{1}{362880} + \frac{1}{4032}\beta + \frac{143}{55296}\beta^2 + \frac{1321}{138240}\beta^3 + \frac{6799}{737280}\beta^4\right)\phi^9$$
$$- \left(\frac{1}{39916800} + \frac{103}{14515200}\beta + \frac{565}{2322432}\beta^2 + \frac{3937}{2211840}\beta^3 + \frac{121447}{22118400}\beta^4 + \frac{443821}{88473600}\beta^5\right)\phi^{11}$$

to the 11th order, and

$$z=\frac{1}{2}\phi^2-\left(\frac{1}{24}+\frac{3}{32}\beta\right)\phi^4+\left(\frac{1}{720}+\frac{1}{72}\beta+\frac{5}{144}\beta^2\right)\phi^6$$
$$-\left(\frac{1}{40320}+\frac{49}{46080}\beta+\frac{67}{9216}\beta^2+\frac{1183}{73728}\beta^3\right)\phi^8$$
$$+\left(\frac{1}{3628800}+\frac{11}{241920}\beta+\frac{157}{184320}\beta^2+\frac{2987}{691200}\beta^3+\frac{6799}{819200}\beta^4\right)\phi^{10}$$
$$-\left(\frac{1}{479001600}+\frac{269}{174182400}\beta+\frac{7993}{139345920}\beta^2+\frac{3551}{5308416}\beta^3\right.$$
$$\left.+\frac{724007}{265420800}\beta^4+\frac{4882031}{1061683200}\beta^5\right)\phi^{12}$$

to the 12th order.

It is hardly necessary to remark that in these expressions the coefficient of each power of ϕ thus found is exact, and not merely approximate.

Also if s denote the length of the arc of the curve measured from the origin,

$$s=\int\rho d\phi=\phi-\frac{1}{8}\beta\phi^3+\left(\frac{1}{240}\beta+\frac{1}{24}\beta^2\right)\phi^5-\left(\frac{11}{40320}\beta+\frac{3}{896}\beta^2+\frac{169}{9216}\beta^3\right)\phi^7$$
$$+\left(-\frac{1}{80640}\beta+\frac{53}{165888}\beta^2+\frac{2011}{829440}\beta^3+\frac{6799}{737280}\beta^4\right)\phi^9$$
$$-\left(\frac{233}{159667200}\beta-\frac{1}{399168}\beta^2+\frac{1469}{4866048}\beta^3+\frac{104513}{60825600}\beta^4\right.$$
$$\left.+\frac{443821}{88473600}\beta^5\right)\phi^{11}$$

to the 11th order in ϕ.

In order that the terms in these series which involve higher powers of ϕ may be insignificant, ϕ must not exceed a certain limiting value which will, of course, depend on the value of β. The larger the value of β, the smaller will be this limiting value of ϕ.

To find the values of the coordinates for larger values of ϕ, we must proceed step by step according to the method described above, ϕ being taken for t, and x and z in turn taken for y, the value of ϕ being increased at each step by a given small quantity.

Let ω be the circular measure of the interval between two consecutive values of ϕ, then ω must be so chosen that the series above found will give sufficiently accurate values of the coordinates throughout several, say four or five such intervals.

Suppose $\ldots\phi_{-5}, \phi_{-4}, \phi_{-3}, \phi_{-2}, \phi_{-1}, \phi_0$ to be a series of consecutive values of ϕ, with the common difference ω, and let

$$\ldots x_{-5},\ x_{-4},\ x_{-3},\ x_{-2},\ x_{-1},\ x_0$$
$$\ldots z_{-5},\ z_{-4},\ z_{-3},\ z_{-2},\ z_{-1},\ z_0$$

be the corresponding values of the coordinates, and

$$\ldots\rho_{-5},\ \rho_{-4},\ \rho_{-3},\ \rho_{-2},\ \rho_{-1},\ \rho_0$$

the corresponding radii of curvature.

The equations to be integrated are

$$\frac{dx}{d\phi} = \rho \cos\phi,$$

$$\frac{dz}{d\phi} = \rho \sin\phi,$$

where

$$\frac{1}{\rho} + \frac{\sin\phi}{x} = 2 + \beta z.$$

Suppose that the values of the coordinates, and consequently those of the radius of curvature, have been calculated for the successive values of ϕ up to ϕ_{-1}, and we wish to find the values of the same quantities for $\phi = \phi_0$.

In the first place, we may obtain an approximate value of ρ_0 in the following manner.

Tabulate the calculated values of $\log\rho$, and form their successive differences according to the following scheme:

$$\begin{array}{llllll}
\ldots\ldots & & & & & \\
 & \ldots\ldots & & & & \\
\log\rho_{-5} & & \ldots\ldots & & & \\
 & \Delta\log\rho_{-4} & & \Delta^3\log\rho_{-3} & & \\
\log\rho_{-4} & & \Delta^2\log\rho_{-3} & & \Delta^4\log\rho_{-2} & \\
 & \Delta\log\rho_{-3} & & \Delta^3\log\rho_{-2} & & \ldots\ldots \\
\log\rho_{-3} & & \Delta^2\log\rho_{-2} & & \Delta^4\log\rho_{-1} & \\
 & \Delta\log\rho_{-2} & & \Delta^3\log\rho_{-1} & & \\
\log\rho_{-2} & & \Delta^2\log\rho_{-1} & & & \\
 & \Delta\log\rho_{-1} & & & & \\
\log\rho_{-1} & & & & &
\end{array}$$

If ω is taken sufficiently small, the differences as we proceed to higher orders will rapidly diminish, and it will generally be easy by inspection of the two or three last calculated fourth differences, to fix upon an approximate value of the fourth difference $\Delta^4\log\rho_0$ immediately succeeding.

Call this approximate value $\Delta^4\log(\rho_0)$, and by successive additions form $\Delta^3\log(\rho_0)$, $\Delta^2\log(\rho_0)$, $\Delta\log(\rho_0)$ and $\log(\rho_0)$, thus

$$\begin{array}{lllll}
 & & & & \Delta^4\log\rho_{-1} \\
 & & & \Delta^3\log\rho_{-1} & \\
 & & \Delta^2\log\rho_{-1} & & \Delta^4\log(\rho_0) \\
 & \Delta\log\rho_{-1} & & \Delta^3\log(\rho_0) & \\
\log\rho_{-1} & & \Delta^2\log(\rho_0) & & \\
 & \Delta\log(\rho_0) & & & \\
\log(\rho_0) & & & &
\end{array}$$

Form the values of

$$\ldots dx_{-5},\ dx_{-4},\ dx_{-3},\ dx_{-2},\ dx_{-1},$$
$$\ldots dz_{-5},\ dz_{-4},\ dz_{-3},\ dz_{-2},\ dz_{-1},$$

and of their successive differences, according to the following scheme:

$$\begin{array}{llllll}
\cdots\cdots \quad \cdots\cdots & & & & & \\
 & \cdots\cdots & & \cdots\cdots & & \\
\omega\rho_{-5}\cos\phi_{-5}=dx_{-5} & & \Delta^2 dx_{-4} & & \Delta^4 dx_{-3} & \\
 & \Delta dx_{-4} & & \Delta^3 dx_{-3} & & \cdots\cdots \\
\omega\rho_{-4}\cos\phi_{-4}=dx_{-4} & & \Delta^2 dx_{-3} & & \Delta^4 dx_{-2} & \\
 & \Delta dx_{-3} & & \Delta^3 dx_{-2} & & \cdots\cdots \\
\omega\rho_{-3}\cos\phi_{-3}=dx_{-3} & & \Delta^2 dx_{-2} & & \Delta^4 dx_{-1} & \\
 & \Delta dx_{-2} & & \Delta^3 dx_{-1} & & \\
\omega\rho_{-2}\cos\phi_{-2}=dx_{-2} & & \Delta^2 dx_{-1} & & & \\
 & \Delta dx_{-1} & & & & \\
\omega\rho_{-1}\cos\phi_{-1}=dx_{-1} & & & & &
\end{array}$$

and

$$\begin{array}{llllll}
\cdots\cdots \quad \cdots\cdots & & & & & \\
 & \cdots\cdots & & \cdots\cdots & & \\
\omega\rho_{-5}\sin\phi_{-5}=dz_{-5} & & \Delta^2 dz_{-4} & & \Delta^4 dz_{-3} & \\
 & \Delta dz_{-4} & & \Delta^3 dz_{-3} & & \cdots\cdots \\
\omega\rho_{-4}\sin\phi_{-4}=dz_{-4} & & \Delta^2 dz_{-3} & & \Delta^4 dz_{-2} & \\
 & \Delta dz_{-3} & & \Delta^3 dz_{-2} & & \cdots\cdots \\
\omega\rho_{-3}\sin\phi_{-3}=dz_{-3} & & \Delta^2 dz_{-2} & & \Delta^4 dz_{-1} & \\
 & \Delta dz_{-2} & & \Delta^3 dz_{-1} & & \\
\omega\rho_{-2}\sin\phi_{-2}=dz_{-2} & & \Delta^2 dz_{-1} & & & \\
 & \Delta dz_{-1} & & & & \\
\omega\rho_{-1}\sin\phi_{-1}=dz_{-1} & & & & &
\end{array}$$

If ρ_0 were known, we might similarly form

$$dx_0=\omega\rho_0\cos\phi_0, \qquad \text{and } dz_0=\omega\rho_0\sin\phi_0,$$

and the successive differences

$$\Delta dx_0,\ \Delta^2 dx_0,\ \Delta^3 dx_0,\ \Delta^4 dx_0,\ \&c.,$$
$$\Delta dz_0,\ \Delta^2 dz_0,\ \Delta^3 dz_0,\ \Delta^4 dz_0,\ \&c.,$$

and then we should have, by what has been already proved,

$$x_0-x_{-1}=dx_0-\frac{1}{2}\Delta dx_0-\frac{1}{12}\Delta^2 dx_0-\frac{1}{24}\Delta^3 dx_0-\frac{19}{720}\Delta^4 dx_0-\&c.,$$

and $$z_0-z_{-1}=dz_0-\frac{1}{2}\Delta dz_0-\frac{1}{12}\Delta^2 dz_0-\frac{1}{24}\Delta^3 dz_0-\frac{19}{720}\Delta^4 dz_0-\&c.;$$

and when x_0 and z_0 had thus been found, we should have the equation

$$\frac{1}{\rho_0}+\frac{\sin\phi_0}{x_0}=2+\beta z_0$$

in verification of the value which had been used for ρ_0.

Now, let (dx_0) and (dz_0) be approximate values of dx_0 and dz_0 respectively, given by

$$(dx_0) = \omega(\rho_0)\cos\phi_0,$$

$$(dz_0) = \omega(\rho_0)\sin\phi_0,$$

and let the successive differences found by employing (dx_0) instead of dx_0, and (dz_0) instead of dz_0, be denoted by

$$\Delta(dx_0),\quad \Delta^2(dx_0),\quad \Delta^3(dx_0),\quad \Delta^4(dx_0),\ \&c.,$$

and

$$\Delta(dz_0),\quad \Delta^2(dz_0),\quad \Delta^3(dz_0),\quad \Delta^4(dz_0),\ \&c.,$$

respectively, and suppose that (x_0) and (z_0) are given by the equations

$$(x_0) - x_{-1} = (dx_0) - \frac{1}{2}\Delta(dx_0) - \frac{1}{12}\Delta^2(dx_0) - \frac{1}{24}\Delta^3(dx_0) - \frac{19}{720}\Delta^4(dx_0) - \&c.,$$

$$(z_0) - z_{-1} = (dz_0) - \frac{1}{2}\Delta(dz_0) - \frac{1}{12}\Delta^2(dz_0) - \frac{1}{24}\Delta^3(dz_0) - \frac{19}{720}\Delta^4(dz_0) - \&c.$$

Also let $[\rho_0]$ be found from the equation

$$\frac{1}{[\rho_0]} + \frac{\sin\phi_0}{(x_0)} = 2 + \beta(z_0),$$

and suppose that this gives $\qquad [\rho_0] = (\rho_0)(1+\epsilon),$

where ϵ is a very small known quantity.

Then if the true value of $\rho_0 = (\rho_0)(1+\eta)$, the correction of the value of (dx_0), and therefore also that of the values of $\Delta(dx_0)$, $\Delta^2(dx_0)$, $\Delta^3(dx_0)$, $\Delta^4(dx_0)$, &c. will be

$$\eta\omega(\rho_0)\cos\phi_0,$$

and the correction of the values of (dz_0), $\Delta(dz_0)$, $\Delta^2(dz_0)$, $\Delta^3(dz_0)$, $\Delta^4(dz_0)$, &c. will be

$$\eta\omega(\rho_0)\sin\phi_0.$$

Hence if we stop at the terms which involve differences of the 4th order, we shall have

$$x_0 - (x_0) = \frac{251}{720}\eta\omega(\rho_0)\cos\phi_0,$$

and

$$z_0 - (z_0) = \frac{251}{720}\eta\omega(\rho_0)\sin\phi_0.$$

Hence, since

$$\frac{1}{\rho_0} + \frac{\sin\phi_0}{x_0} = 2 + \beta z_0$$

and

$$\frac{1}{[\rho_0]} + \frac{\sin\phi_0}{(x_0)} = 2 + \beta(z_0),$$

we find

$$\frac{1}{\rho_0} - \frac{1}{[\rho_0]} = -\sin\phi_0\left[\frac{1}{x_0} - \frac{1}{(x_0)}\right] + \beta[z_0 - (z_0)]$$

$$= \frac{251}{720}\eta\omega(\rho_0)\sin\phi_0\left[\frac{\cos\phi_0}{(x_0)^2} + \beta\right] \text{ nearly};$$

but $$\frac{1}{\rho_0} = \frac{1}{(\rho_0)}(1-\eta), \text{ nearly,}$$

and $$\frac{1}{[\rho_0]} = \frac{1}{(\rho_0)}(1-\epsilon), \text{ nearly,}$$

Hence $$\frac{1}{(\rho_0)}[\epsilon-\eta] = \frac{251}{720}\eta\omega(\rho_0)\sin\phi_0\left[\frac{\cos\phi_0}{(x_0)^2}+\beta\right], \text{ nearly,}$$

and therefore $$\eta = \frac{\epsilon}{1+\frac{251}{720}\omega(\rho_0)^2\sin\phi_0\left[\frac{\cos\phi_0}{(x_0)^2}+\beta\right]}, \text{ nearly.}$$

Hence η is found, and therefore the values of x_0 and z_0, which were required, become known.

In practice, the following slight modification of the above process will be found convenient.

Suppose the assumed value of $\log(\rho_0)$ to be increased by 100 units of the last place of decimals employed, then while calculating the values of (dx_0), (dz_0), (x_0), (z_0) and the consequent value of $[\rho_0]$, note at the side of the work, the changes which would be severally caused in each of these quantities by such an augmentation of $\log(\rho_0)$. It may be remarked that the changes in (x_0) and (z_0) will be $\frac{251}{720}$ times the changes in (dx_0) and (dz_0) respectively, when we stop at terms involving Δ^4, and that $\frac{251}{720}$ may be conveniently put under the form

$$\frac{1}{3}\left[1+\frac{1}{20}\left(1-\frac{1}{12}\right)\right].$$

Now suppose that an increase of 100 units in $\log(\rho_0)$ causes a diminution of μ units in $\log[\rho_0]$, and that the excess of $\log[\rho_0]$ above $\log(\rho_0)$ is λ of the same units, then the correction to be applied to the assumed value $\log(\rho_0)$ will be

$$\lambda\frac{100}{100+\mu} \text{ such units,}$$

and the correction to the value of $\log[\rho_0]$ will be

$$-\frac{\lambda\mu}{100+\mu} \text{ such units,}$$

and the proportionate changes required in the values of (dx_0), (dz_0), (x_0) and (z_0) will be at once found.

If in finding (x_0) and (z_0) we include the terms which involve differences of the 5th order, the fraction $\frac{251}{720}$, which occurs in the above, should be replaced by $\frac{95}{288} = \frac{1}{3}\left(1-\frac{1}{96}\right)$.

We may, of course, change the value of ω whenever the more or less rapid rate of diminution of the successive differences shews that it is expedient to increase or diminish the interval. It is only necessary, by selection from or interpolation between the values already calculated, to find the coordinates for a few values of ϕ separated from each other by the newly chosen interval.

It may be remarked that when, by means of the appropriate series, we have found the values of ρ for a sufficient number of small values of ϕ, we can form the corresponding values of dx and dz, and thence derive the corresponding values of x and z by the process of integration before explained, without the necessity of employing the series for finding those quantities.

A numerical example of the work will be given later on.

When x and z, and therefore also ρ, are known for a given value of ϕ, we can find the volume of the portion of the drop terminated by the horizontal plane at distance z from the origin, without any further integration.

For if V be this volume, we have

$$dV = \pi x^2 dz.$$

Also from the differential equation, when the radius of curvature at the vertex is taken for the unit of length,

$$\frac{1}{\rho} + \frac{\sin\phi}{x} = 2 + \beta z,$$

and therefore
$$\beta dz = -\frac{d\rho}{\rho^2} + \frac{\cos\phi\, d\phi}{x} - \frac{\sin\phi\, dx}{x^2}.$$

Hence
$$dV = \frac{\pi}{\beta}\left\{-\frac{x^2 d\rho}{\rho^2} + x\cos\phi\, d\phi - \sin\phi\, dx\right\},$$

and, integrating the first term of dV by parts,

$$V = \frac{\pi}{\beta}\left\{\frac{x^2}{\rho} - 2\int\frac{x dx}{\rho} + \int x\cos\phi\, d\phi - \int\sin\phi\, dx\right\}$$

$$= \frac{\pi}{\beta}\left\{\frac{x^2}{\rho} - \int(x\cos\phi\, d\phi + \sin\phi\, dx)\right\},$$

since
$$dx = \rho\cos\phi\, d\phi,$$

or
$$V = \frac{\pi}{\beta}\left\{\frac{x^2}{\rho} - x\sin\phi\right\} = \frac{\pi x^2}{\beta}\left\{\frac{1}{\rho} - \frac{\sin\phi}{x}\right\},$$

no correction being required, since V vanishes with x.

If now the radius of curvature at the vertex be equal to b instead of being unity, we must replace x and ρ by $\frac{x}{b}$ and $\frac{\rho}{b}$ respectively, and V by $\frac{V}{b^3}$.

Hence we have, in this case,

$$V = \frac{\pi b^2 x^2}{\beta}\left\{\frac{1}{\rho} - \frac{\sin\phi}{x}\right\};$$

or, replacing $\frac{1}{\rho}$ by its equivalent $\frac{2}{b} + \frac{\beta z}{b^2} - \frac{\sin\phi}{x}$,

$$V = \frac{\pi b^2 x^2}{\beta}\left\{\frac{2}{b} - \frac{2\sin\phi}{x} + \frac{\beta z}{b^2}\right\}.$$

The expression just found for V is, I believe, due to Bertrand, who gives it in Liouville's Journal, Tome XIII. p. 185.

This result may also be found very simply, without employing the differential equation, thus:

The portion of the drop of which the volume is V is kept in equilibrium by

(1) its own weight,

(2) the pressure of the contiguous fluid on the opposite side of the bounding plane,

(3) the tension across the circle in which the bounding plane meets the surface of the fluid.

This tension acts at each point in a direction which makes the same angle ϕ with the horizontal plane. Hence if T be the tension per unit of length, since $2\pi x$ is the circumference of the circle before mentioned, the resolved part of the tension in a vertical direction is

$$2\pi x T \sin \phi.$$

Also the weight is $g\sigma V$, and the total pressure of the contiguous fluid is

$$\pi x^2 p,$$

where p is the pressure at any point of the bounding plane, so that

$$p = T\left(\frac{1}{\rho} + \frac{\sin \phi}{x}\right).$$

Hence $$g\sigma V + 2\pi x T \sin \phi = \pi x^2 T \left(\frac{1}{\rho} + \frac{\sin \phi}{x}\right),$$

and $$V = \frac{\pi x^2 T}{g\sigma}\left(\frac{1}{\rho} - \frac{\sin \phi}{x}\right),$$

which is identical with the former result, since as we have seen

$$\beta = \frac{g\sigma b^2}{T}.$$

If the constant β be positive, the process above explained, in which ϕ is taken as the independent variable, may be carried on to any extent that may be desired, and the radius of curvature ρ of the meridional section of the drop will always remain finite. But if β be negative, that is if the drop hang from the lower side of a horizontal plane, then if we proceed further and further from the vertex, the value of $\frac{1}{\rho}$, which at the vertex itself is supposed to be unity, will continually diminish, until at a certain point it vanishes and then becomes negative. The point of the curve at which ρ thus changes sign by passing through infinity is a point of inflexion, where ϕ attains a maximum value. Hence all differential coefficients taken with respect to ϕ as the independent variable become infinite at the point in question and have large positive or negative values at points near it.

This circumstance makes it necessary, when β is negative, to choose a different independent variable.

Suppose now that s, the length of the arc measured from the vertex, is taken as the independent variable.

The equations to be integrated are

$$d\phi = \frac{1}{\rho}\, ds,$$

$$dx = \cos\phi\, ds,$$

$$dz = \sin\phi\, ds,$$

where the value of $\frac{1}{\rho}$ in terms of x, z and ϕ is given, as before, by the equation

$$\frac{1}{\rho} + \frac{\sin\phi}{x} = 2 + \beta z.$$

Also to determine the constants of integration, we have, when $s = 0$,

$$x = 0,\ \ z = 0,\ \ \phi = 0, \text{ and } \frac{1}{\rho} = 1.$$

We must first find the form of the curve in the neighbourhood of the vertex, by developing ϕ, x and z in series of ascending powers of s.

We have already found s as well as x and z in series of ascending powers of ϕ, and by means of Lagrange's theorem it is easy to transform these series so as to obtain the required series in powers of s.

From the expression of s in terms of ϕ, we find by transposition,

$$\begin{aligned}\phi = s &+ \frac{1}{8}\beta\phi^3 - \left(\frac{1}{240}\beta + \frac{1}{24}\beta^2\right)\phi^5 + \left(\frac{11}{40320}\beta + \frac{3}{896}\beta^2 + \frac{169}{9216}\beta^3\right)\phi^7 \\ &- \left(-\frac{1}{80640}\beta + \frac{53}{165888}\beta^2 + \frac{2011}{829440}\beta^3 + \frac{6799}{737280}\beta^4\right)\phi^9 \\ &+ \left(\frac{233}{159667200}\beta - \frac{1}{399168}\beta^2 + \frac{1469}{4866048}\beta^3 + \frac{104513}{60825600}\beta^4 + \frac{443821}{88473600}\beta^5\right)\phi^{11} \\ &- \&c.,\end{aligned}$$

or $\quad \phi = s + F(\phi)$, suppose,

which is in the proper form for the application of Lagrange's theorem.

Hence, we have

$$\phi = s + F(s) + \frac{1}{1.2}\frac{d}{ds}[F(s)]^2 + \frac{1}{1.2.3}\frac{d^2}{ds^2}[F(s)]^3 + \frac{1}{1.2.3.4}\frac{d^3}{ds^3}[F(s)]^4 + \&c.$$

Also if in the values of x, z, $\frac{dx}{d\phi}$, $\frac{dz}{d\phi}$ in terms of ϕ, we change ϕ into s, and denote the results by

$$(x),\ (z),\ \left(\frac{dx}{d\phi}\right) \text{ and } \left(\frac{dz}{d\phi}\right),$$

we have, by the same theorem,

$$x = (x) + \left(\frac{dx}{d\phi}\right) F(s) + \frac{1}{1.2}\frac{d}{ds}\left[\left(\frac{dx}{d\phi}\right) F(s)\right] + \frac{1}{1.2.3}\frac{d^2}{ds^2}\left[\left(\frac{dx}{d\phi}\right)[F(s)]^2\right] + \&c.,$$

$$z = (z) + \left(\frac{dz}{d\phi}\right) F(s) + \frac{1}{1.2}\frac{d}{ds}\left[\left(\frac{dz}{d\phi}\right) F(s)\right] + \frac{1}{1.2.3}\frac{d^2}{ds^2}\left[\left(\frac{dz}{d\phi}\right)[F(s)]^2\right] + \&c.$$

In this way, we obtain

$$\phi = s + \frac{1}{8}\beta s^3 + \left(-\frac{1}{240}\beta + \frac{1}{192}\beta^2\right)s^5 + \left(\frac{11}{40320}\beta - \frac{11}{13440}\beta^2 + \frac{1}{9216}\beta^3\right)s^7$$
$$+ \left(\frac{1}{80640}\beta + \frac{629}{5806080}\beta^2 - \frac{487}{5806080}\beta^3 + \frac{1}{737280}\beta^4\right)s^9$$
$$+ \left(\frac{233}{159667200}\beta + \frac{7}{2851200}\beta^2 + \frac{17539}{851558400}\beta^3 - \frac{271}{47308800}\beta^4 + \frac{1}{88473600}\beta^5\right)s^{11}$$
$$+ \&c., \&c.$$

$$x = s - \frac{1}{6}s^3 + \left(\frac{1}{120} - \frac{1}{40}\beta\right)s^5 - \left(\frac{1}{5040} - \frac{1}{280}\beta + \frac{5}{2688}\beta^2\right)s^7$$
$$+ \left(\frac{1}{362880} - \frac{1}{4480}\beta + \frac{493}{725760}\beta^2 - \frac{7}{82944}\beta^3\right)s^9$$
$$- \left(\frac{1}{39916800} - \frac{1}{118800}\beta + \frac{26617}{319334400}\beta^2 - \frac{1273}{15966720}\beta^3 + \frac{7}{2703360}\beta^4\right)s^{11}$$
$$+ \&c., \&c.$$

$$z = \frac{1}{2}s^2 + \left(-\frac{1}{24} + \frac{1}{32}\beta\right)s^4 + \left(\frac{1}{720} - \frac{1}{90}\beta + \frac{1}{1152}\beta^2\right)s^6$$
$$+ \left(-\frac{1}{40320} + \frac{61}{64512}\beta - \frac{151}{107520}\beta^2 + \frac{1}{73728}\beta^3\right)s^8$$
$$+ \left(\frac{1}{3628800} - \frac{19}{403200}\beta + \frac{14849}{58060800}\beta^2 - \frac{809}{7257600}\beta^3 + \frac{1}{7372800}\beta^4\right)s^{10}$$
$$+ \left(-\frac{1}{479001600} + \frac{2477}{1916006400}\beta - \frac{2917}{121651200}\beta^2 + \frac{1264267}{30656102400}\beta^3 - \frac{42137}{6812467200}\beta^4 + \frac{1}{1061683200}\beta^5\right)s^{12}$$
$$+ \&c., \&c.$$

Also, we have

$$\frac{1}{\rho}=\frac{d\phi}{ds}$$

$$=1+\frac{3}{8}\beta s^2+\left(-\frac{1}{48}\beta+\frac{5}{192}\beta^2\right)s^4+\left(\frac{11}{5760}\beta-\frac{11}{1920}\beta^2+\frac{7}{9216}\beta^3\right)s^6$$

$$+\left(\frac{1}{8960}\beta+\frac{629}{645120}\beta^2-\frac{487}{645120}\beta^3+\frac{1}{81920}\beta^4\right)s^8$$

$$+\left(\frac{233}{14515200}\beta+\frac{7}{259200}\beta^2+\frac{17539}{77414400}\beta^3-\frac{271}{4300800}\beta^4+\frac{11}{88473600}\beta^5\right)s^{10}$$

$$+\ \&c.,\ \&c.$$

As before, it may be remarked that the coefficient of each power of s thus found is exact, and not merely approximate.

We may also find these series for ϕ, x and z in terms of s independently, in the following manner:

Assume, as we evidently may do,

$$\frac{1}{\rho}=1+c_2s^2+c_4s^4+c_6s^6+c_8s^8+c_{10}s^{10}+\&c.,$$

therefore
$$\phi=\int\frac{ds}{\rho}=s+\frac{1}{3}c_2s^3+\frac{1}{5}c_4s^5+\frac{1}{7}c_6s^7+\frac{1}{9}c_8s^9+\frac{1}{11}c_{10}s^{11}+\&c.,$$

since ϕ and s vanish together.

Hence we may find

$$\cos\phi=1-\frac{1}{2}s^2+\left(\frac{1}{24}-\frac{1}{3}c_2\right)s^4-\left(\frac{1}{720}-\frac{1}{18}c_2+\frac{1}{18}c_2^2+\frac{1}{5}c_4\right)s^6$$

$$+\left(\frac{1}{40320}-\frac{1}{360}c_2+\frac{1}{36}c_2^2+\frac{1}{30}c_4-\frac{1}{15}c_2c_4-\frac{1}{7}c_6\right)s^8$$

$$-\left(\frac{1}{3628800}-\frac{1}{15120}c_2+\frac{1}{432}c_2^2-\frac{1}{162}c_2^3+\frac{1}{600}c_4-\frac{1}{30}c_2c_4\right.$$
$$\left.+\frac{1}{50}c_4^2-\frac{1}{42}c_6+\frac{1}{21}c_2c_6+\frac{1}{9}c_8\right)s^{10}$$

$$+\ \&c.,\ \&c.,$$

and

$$\sin\phi=s-\left(\frac{1}{6}-\frac{1}{3}c_2\right)s^3+\left(\frac{1}{120}-\frac{1}{6}c_2+\frac{1}{5}c_4\right)s^5-\left(\frac{1}{5040}-\frac{1}{72}c_2+\frac{1}{18}c_2^2+\frac{1}{10}c_4-\frac{1}{7}c^5\right)s^7$$

$$+\left(\frac{1}{362880}-\frac{1}{2160}c_2+\frac{1}{108}c_2^2-\frac{1}{162}c_2^3+\frac{1}{120}c_4-\frac{1}{15}c_2c_4-\frac{1}{14}c_6+\frac{1}{9}c_8\right)s^9$$

$$-\left(\frac{1}{39916800}-\frac{1}{120960}c_2+\frac{1}{2160}c_2^2-\frac{1}{324}c_2^3+\frac{1}{3600}c_4-\frac{1}{90}c_2c_4\right.$$
$$\left.+\frac{1}{90}c_2^2c_4+\frac{1}{50}c_4^2-\frac{1}{168}c_6+\frac{1}{21}c_2c_6+\frac{1}{18}c_8-\frac{1}{11}c_{10}\right)s^{11}$$

$$+\ \&c.,\ \&c.$$

And therefore

$$x=\int\cos\phi\,ds=s-\frac{1}{6}s^3+\left(\frac{1}{120}-\frac{1}{15}c_2\right)s^5-\left(\frac{1}{5040}-\frac{1}{126}c_2+\frac{1}{126}c_2^{\,2}+\frac{1}{35}c_4\right)s^7$$

$$+\left(\frac{1}{362880}-\frac{1}{3240}c_2+\frac{1}{324}c_2^{\,2}+\frac{1}{270}c_4-\frac{1}{135}c_2c_4-\frac{1}{63}c_6\right)s^9$$

$$-\left(\frac{1}{39916800}-\frac{1}{166320}c_2+\frac{1}{4752}c_2^{\,2}-\frac{1}{1782}c_2^{\,3}+\frac{1}{6600}c_4-\frac{1}{330}c_2c_4\right.$$

$$\left.+\frac{1}{550}c_4^{\,2}-\frac{1}{462}c_6+\frac{1}{231}c_2c_6+\frac{1}{99}c_8\right)s^{11}$$

$+$ &c., &c.,

and

$$z=\int\sin\phi\,ds=\frac{1}{2}s^2-\left(\frac{1}{24}-\frac{1}{12}c_2\right)s^4+\left(\frac{1}{720}-\frac{1}{36}c_2+\frac{1}{30}c_4\right)s^6$$

$$-\left(\frac{1}{40320}-\frac{1}{576}c_2+\frac{1}{144}c_2^{\,2}+\frac{1}{80}c_4-\frac{1}{56}c_6\right)s^8$$

$$+\left(\frac{1}{3628800}-\frac{1}{21600}c_2+\frac{1}{1080}c_2^{\,2}-\frac{1}{1620}c_2^{\,3}+\frac{1}{1200}c_4-\frac{1}{150}c_2c_4\right.$$

$$\left.-\frac{1}{140}c_6+\frac{1}{90}c_8\right)s^{10}$$

$$-\left(\frac{1}{479001600}-\frac{1}{1451520}c_2+\frac{1}{25920}c_2^{\,2}-\frac{1}{3888}c_2^{\,3}+\frac{1}{43200}c_4\right.$$

$$-\frac{1}{1080}c_2c_4+\frac{1}{1080}c_2^{\,2}c_4+\frac{1}{600}c_4^{\,2}-\frac{1}{2016}c_6+\frac{1}{252}c_2c_6$$

$$\left.+\frac{1}{216}c_8-\frac{1}{132}c_{10}\right)s^{12}$$

$+$ &c., &c.

Hence, we may find by division

$$\frac{\sin\phi}{x}=1+\frac{1}{3}c_2s^2+\left(-\frac{2}{45}c_2+\frac{1}{5}c_4\right)s^4+\left(-\frac{4}{945}c_2-\frac{8}{315}c_2^{\,2}-\frac{4}{105}c_4+\frac{1}{7}c_6\right)s^6$$

$$+\left(-\frac{2}{4725}c_2-\frac{52}{14175}c_2^{\,2}-\frac{2}{567}c_2^{\,3}-\frac{16}{4725}c_4-\frac{172}{4725}c_2c_4-\frac{2}{63}c_6+\frac{1}{9}c_8\right)s^8$$

$$+\left(-\frac{4}{93555}c_2-\frac{32}{66825}c_2^{\,2}-\frac{532}{467775}c_2^{\,3}-\frac{52}{155925}c_4-\frac{16}{3465}c_2c_4-\frac{4}{567}c_2^{\,2}c_4\right.$$

$$\left.-\frac{24}{1925}c_4^{\,2}-\frac{4}{1485}c_6-\frac{296}{10395}c_2c_6-\frac{8}{297}c_8+\frac{1}{11}c_{10}\right)s^{10}$$

$+$ &c., &c.

Substitute these expressions in the equation

$$\frac{1}{\rho} + \frac{\sin\phi}{x} = 2 + \beta z,$$

and equate the coefficients of corresponding powers of s, and we shall find successively,

$$c_2 = \frac{3}{8}\beta,$$

$$c_4 = -\frac{1}{48}\beta + \frac{5}{192}\beta^2,$$

$$c_6 = \frac{11}{5760}\beta - \frac{11}{1920}\beta^2 + \frac{7}{9216}\beta^3,$$

$$c_8 = \frac{1}{8960}\beta + \frac{629}{645120}\beta^2 - \frac{487}{645120}\beta^3 + \frac{1}{81920}\beta^4,$$

$$c_{10} = \frac{233}{14515200}\beta + \frac{7}{259200}\beta^2 + \frac{17539}{77414400}\beta^3 - \frac{271}{4300800}\beta^4 + \frac{11}{88473600}\beta^5,$$

which agree with the coefficients of the several powers of s in the value of $\frac{1}{\rho}$ which has been already found in another way.

Also by the substitution of these coefficients in the expressions for x and z given above, we shall obtain the same values of x and z as those which have been before found.

By means of the above series, we may determine the values of x, z and ϕ for given values of s in the neighbourhood of the origin. As in the case where ϕ was taken as the independent variable, in order that the terms of these series which involve higher powers of s may be insignificant, s must not exceed a certain limiting value which will, of course, depend on the value of β. The larger the value of β, the smaller will be this limiting value of s.

In order to find the values of x, z and ϕ for larger values of s, we must proceed step by step, as in the former case, the value of s being increased at each step by a given small quantity, ω suppose.

The interval ω should be so chosen that the series above found will give sufficiently accurate values of x, z and ϕ throughout several, say four or five such intervals.

The process to be followed is exactly similar to that explained before, except that in this case there are three quantities x, z and ϕ to be determined by integration instead of the two x and z.

It is this circumstance only which makes it preferable to employ ϕ as the independent variable in the case where this method is applicable, viz. when β is a positive quantity.

The present method is equally applicable whether β be positive or negative.

Now, suppose $\ldots s_{-5}, s_{-4}, s_{-3}, s_{-2}, s_{-1}, s_0$ to be a series of consecutive values of s, with the common difference ω, and let $\ldots \phi_{-5}, \phi_{-4}, \phi_{-3}, \phi_{-2}, \phi_{-1}, \phi_0$ be the corresponding values of ϕ, and

$$\ldots x_{-5},\ x_{-4},\ x_{-3},\ x_{-2},\ x_{-1},\ x_0,$$

$$\ldots z_{-5},\ z_{-4},\ z_{-3},\ z_{-2},\ z_{-1},\ z_0,$$

the corresponding values of the coordinates, and

$$\ldots \rho_{-5},\ \rho_{-4},\ \rho_{-3},\ \rho_{-2},\ \rho_{-1},\ \rho_0,$$

the corresponding radii of curvature.

The equations to be integrated are

$$\frac{d\phi}{ds} = \frac{1}{\rho},$$

$$\frac{dx}{ds} = \cos\phi,$$

$$\frac{dz}{ds} = \sin\phi,$$

where

$$\frac{1}{\rho} + \frac{\sin\phi}{x} = 2 + \beta z.$$

Suppose that the values of ϕ, x and z, and consequently also the corresponding values of the radius of curvature ρ, are known for the successive values of s up to s_{-1}, and we wish to find the value of each of these quantities for $s = s_0$.

In the first place, we may obtain an approximate value of $\frac{1}{\rho_0}$ in the following manner.

Tabulate the calculated values of $\frac{1}{\rho}$, and form their successive differences according to the following scheme:

$$\begin{array}{cccccc}
\cdots & & \cdots & & & \\
 & \cdots & & \cdots & & \\
\frac{1}{\rho_{-5}} & & \cdots & & \cdots & \\
 & \Delta\frac{1}{\rho_{-4}} & & \Delta^3\frac{1}{\rho_{-3}} & & \\
\frac{1}{\rho_{-4}} & & \Delta^2\frac{1}{\rho_{-3}} & & \Delta^4\frac{1}{\rho_{-2}} & \\
 & \Delta\frac{1}{\rho_{-3}} & & \Delta^3\frac{1}{\rho_{-2}} & & \cdots \\
\frac{1}{\rho_{-3}} & & \Delta^2\frac{1}{\rho_{-2}} & & \Delta^4\frac{1}{\rho_{-1}} & \\
 & \Delta\frac{1}{\rho_{-2}} & & \Delta^3\frac{1}{\rho_{-1}} & & \\
\frac{1}{\rho_{-2}} & & \Delta^2\frac{1}{\rho_{-1}} & & & \\
 & \Delta\frac{1}{\rho_{-1}} & & & & \\
\frac{1}{\rho_{-1}} & & & & &
\end{array}$$

If ω is taken sufficiently small, the differences as we proceed to higher orders will rapidly diminish, and it will generally be easy by inspection of the two or three last calculated fourth differences to fix upon an approximate value of the fourth difference $\Delta^4 \frac{1}{\rho_0}$ immediately succeeding.

Call this approximate value $\Delta^4\left(\frac{1}{\rho_0}\right)$, so that the approximate is distinguished from the true value by being inclosed in a parenthesis, and by successive additions form $\Delta^3\left(\frac{1}{\rho_0}\right)$, $\Delta^2\left(\frac{1}{\rho_0}\right)$, $\Delta\left(\frac{1}{\rho_0}\right)$ and $\left(\frac{1}{\rho_0}\right)$, thus

$$\begin{array}{ccccc}
 & & & \Delta^3\frac{1}{\rho_{-1}} & \\
 & & & & \Delta^4\left(\frac{1}{\rho_0}\right) \\
 & & \Delta^2\frac{1}{\rho_{-1}} & & \\
\frac{1}{\rho_{-2}} & & & \Delta^3\left(\frac{1}{\rho_0}\right) & \\
 & \Delta\frac{1}{\rho_{-1}} & & & \\
 & & \Delta^2\left(\frac{1}{\rho_0}\right) & & \\
\frac{1}{\rho_{-1}} & & & & \\
 & \Delta\left(\frac{1}{\rho_0}\right) & & & \\
\left(\frac{1}{\rho_0}\right) & & & &
\end{array}$$

Form the values of

$$\begin{array}{l}
\ldots\ d\phi_{-5},\ d\phi_{-4},\ d\phi_{-3},\ d\phi_{-2},\ d\phi_{-1}, \\
\ldots\ dx_{-5},\ dx_{-4},\ dx_{-3},\ dx_{-2},\ dx_{-1}, \\
\ldots\ dz_{-5},\ dz_{-4},\ dz_{-3},\ dz_{-2},\ dz_{-1},
\end{array}$$

and of their successive differences, according to the following scheme:

$$\begin{array}{lllll}
\ldots & \ldots & \ldots & & \\
 & \ldots & & \ldots & \\
\frac{\omega}{\rho_{-5}} = d\phi_{-5} & & \Delta^2 d\phi_{-4} & & \Delta^4 d\phi_{-3} \\
 & \Delta d\phi_{-4} & & \Delta^3 d\phi_{-3} & \\
\frac{\omega}{\rho_{-4}} = d\phi_{-4} & & \Delta^2 d\phi_{-3} & & \Delta^4 d\phi_{-2} \\
 & \Delta d\phi_{-3} & & \Delta^3 d\phi_{-2} & \\
\frac{\omega}{\rho_{-3}} = d\phi_{-3} & & \Delta^2 d\phi_{-2} & & \Delta^4 d\phi_{-1} \\
 & \Delta d\phi_{-2} & & \Delta^3 d\phi_{-1} & \\
\frac{\omega}{\rho_{-2}} = d\phi_{-2} & & \Delta^2 d\phi_{-1} & & \\
 & \Delta d\phi_{-1} & & & \\
\frac{\omega}{\rho_{-1}} = d\phi_{-1} & & & &
\end{array}$$

$$\begin{array}{lllll}
\ldots\ldots & \ldots & \ldots & & \ldots \\
 & \ldots & & \ldots & \\
\omega\cos\phi_{-5} = dx_{-5} & & \Delta^2 dx_{-4} & & \Delta^4 dx_{-3} \\
 & \Delta dx_{-4} & & \Delta^3 dx_{-3} & \\
\omega\cos\phi_{-4} = dx_{-4} & & \Delta^2 dx_{-3} & & \Delta^4 dx_{-2} \\
 & \Delta dx_{-3} & & \Delta^3 dx_{-2} & \\
\omega\cos\phi_{-3} = dx_{-3} & & \Delta^2 dx_{-2} & & \Delta^4 dx_{-1} \\
 & \Delta dx_{-2} & & \Delta^3 dx_{-1} & \\
\omega\cos\phi_{-2} = dx_{-2} & & \Delta^2 dx_{-1} & & \\
 & \Delta dx_{-1} & & & \\
\omega\cos\phi_{-1} = dx_{-1} & & & &
\end{array}$$

and

$$\begin{array}{ccccccccc}
\ldots & & & & \ldots & & & & \ldots \\
 & & \ldots & & & \ldots & & & \\
\omega \sin \phi_{-5} = dz_{-5} & & & & \Delta^2 dz_{-4} & & & & \Delta^4 dz_{-3} \\
 & & \Delta dz_{-4} & & & \Delta^3 dz_{-3} & & & \\
\omega \sin \phi_{-4} = dz_{-4} & & & & \Delta^2 dz_{-3} & & & & \Delta^4 dz_{-2} \\
 & & \Delta dz_{-3} & & & \Delta^3 dz_{-2} & & & \\
\omega \sin \phi_{-3} = dz_{-3} & & & & \Delta^2 dz_{-2} & & & & \Delta^4 dz_{-1} \\
 & & \Delta dz_{-2} & & & \Delta^3 dz_{-1} & & & \\
\omega \sin \phi_{-2} = dz_{-2} & & & & \Delta^2 dz_{-1} & & & & \\
 & & \Delta dz_{-1} & & & & & & \\
\omega \sin \phi_{-1} = dz_{-1} & & & & & & & &
\end{array}$$

If $\dfrac{1}{\rho_0}$ and ϕ_0 were known, we might similarly form

$$d\phi_0 = \frac{\omega}{\rho_0}, \quad dx_0 = \omega \cos \phi_0 \text{ and } dz_0 = \omega \sin \phi_0,$$

and the successive differences

$$\Delta d\phi_0,\ \Delta^2 d\phi_0,\ \Delta^3 d\phi_0,\ \Delta^4 d\phi_0,\ \&c.,$$
$$\Delta dx_0,\ \Delta^2 dx_0,\ \Delta^3 dx_0,\ \Delta^4 dx_0,\ \&c.,$$
$$\Delta dz_0,\ \Delta^2 dz_0,\ \Delta^3 dz_0,\ \Delta^4 dz_0,\ \&c.,$$

and then we should have, by what has been already proved,

$$\phi_0 - \phi_{-1} = d\phi_0 - \frac{1}{2}\Delta d\phi_0 - \frac{1}{12}\Delta^2 d\phi_0 - \frac{1}{24}\Delta^3 d\phi_0 - \frac{19}{720}\Delta^4 d\phi_0 - \&c.,$$

$$x_0 - x_{-1} = dx_0 - \frac{1}{2}\Delta dx_0 - \frac{1}{12}\Delta^2 dx_0 - \frac{1}{24}\Delta^3 dx_0 - \frac{19}{720}\Delta^4 dx_0 - \&c.,$$

$$z_0 - z_{-1} = dz_0 - \frac{1}{2}\Delta dz_0 - \frac{1}{12}\Delta^2 dz_0 - \frac{1}{24}\Delta^3 dz_0 - \frac{19}{720}\Delta^4 dz_0 - \&c.;$$

and when ϕ_0, x_0 and z_0 were thus found, ϕ_0 ought to agree with its assumed value, and the values of ϕ_0, x_0 and z_0 should satisfy the equation

$$\frac{1}{\rho_0} + \frac{\sin \phi_0}{x_0} = 2 + \beta z_0,$$

which thus affords a verification of the value which was used for $\dfrac{1}{\rho_0}$.

Now, let $(d\phi_0)$ be an approximate value of $d\phi_0$, given by

$$(d\phi_0) = \omega\left(\frac{1}{\rho_0}\right),$$

and let the successive differences found by employing $(d\phi_0)$ instead of $d\phi_0$ be denoted by

$$\Delta\,(d\phi_0),\ \Delta^2(d\phi_0),\ \Delta^3(d\phi_0),\ \Delta^4(d\phi_0),\ \&c.,$$

and suppose that (ϕ_0) is given by the equation

$$(\phi_0) - \phi_{-1} = (d\phi_0) - \frac{1}{2}\Delta\,(d\phi_0) - \frac{1}{12}\Delta^2(d\phi_0) - \frac{1}{24}\Delta^3(d\phi_0) - \frac{19}{720}\Delta^4(d\phi_0) - \&c.$$

Also, let (dx_0) and (dz_0) be approximate values of dx_0 and dz_0 respectively, given by

$$(dx_0) = \omega \cos(\phi_0),$$
$$(dz_0) = \omega \sin(\phi_0),$$

and let the successive differences found by employing (dx_0) instead of dx_0, and (dz_0) instead of dz_0, be denoted by

$$\Delta\,(dx_0),\ \Delta^2(dx_0),\ \Delta^3(dx_0),\ \Delta^4(dx_0),\ \&c.,$$

and $\Delta\,(dz_0),\ \Delta^2(dz_0),\ \Delta^3(dz_0),\ \Delta^4(dz_0)$, &c., respectively,

and suppose that (x_0) and (z_0) are given by the equations

$$(x_0) - x_{-1} = (dx_0) - \frac{1}{2}\Delta\,(dx_0) - \frac{1}{12}\Delta^2(dx_0) - \frac{1}{24}\Delta^3(dx_0) - \frac{19}{720}\Delta^4(dx_0) - \&c.,$$

$$(z_0) - z_{-1} = (dz_0) - \frac{1}{2}\Delta\,(dz_0) - \frac{1}{12}\Delta^2(dz_0) - \frac{1}{24}\Delta^3(dz_0) - \frac{19}{720}\Delta^4(dz_0) - \&c.$$

Also let $\left[\frac{1}{\rho_0}\right]$ be found from the equation

$$\left[\frac{1}{\rho_0}\right] + \frac{\sin(\phi_0)}{(x_0)} = 2 + \beta\,(z_0),$$

and suppose that this gives

$$\left[\frac{1}{\rho_0}\right] = \left(\frac{1}{\rho_0}\right) + \epsilon,$$

where ϵ is a very small known quantity.

Then, if the true value of $\frac{1}{\rho_0} = \left(\frac{1}{\rho_0}\right) + \eta$, the correction of the value of $(d\phi_0)$, and therefore also that of the values of $\Delta\,(d\phi_0)$, $\Delta^2(d\phi_0)$, $\Delta^3(d\phi_0)$, $\Delta^4(d\phi_0)$, &c., will be $\omega\eta$.

Hence, if we stop at the terms which involve differences of the 4th order, we shall have

$$\phi_0 - (\phi_0) = \frac{251}{720}\,\omega\eta.$$

Wherefore
$$\cos\phi_0 = \cos(\phi_0) - \frac{251}{720}\,\omega\eta\sin(\phi_0)$$

and
$$\sin\phi_0 = \sin(\phi_0) + \frac{251}{720}\,\omega\eta\cos(\phi_0).$$

Hence the correction to be applied to the values of (dx_0), $\Delta\,(dx_0)$, $\Delta^2(dx_0)$, $\Delta^3(dx_0)$, $\Delta^4(dx_0)$, &c., will be

$$-\frac{251}{720}\,\omega^2\eta\sin(\phi_0),$$

and the correction to be applied to the values of (dz_0), $\Delta(dz_0)$, $\Delta^2(dz_0)$, $\Delta^3(dz_0)$, $\Delta^4(dz_0)$, &c., will be

$$\frac{251}{720}\,\omega^2\eta\cos(\phi_0).$$

Whence if, as before, we stop at the terms which involve differences of the 4th order, we shall have

$$x_0 - (x_0) = -\left(\frac{251}{720}\,\omega\right)^2 \eta\sin(\phi_0),$$

$$z_0 - (z_0) = \left(\frac{251}{720}\,\omega\right)^2 \eta\cos(\phi_0).$$

Hence, since

$$\frac{1}{\rho_0} + \frac{\sin\phi_0}{x_0} = 2 + \beta z_0$$

and

$$\left[\frac{1}{\rho_0}\right] + \frac{\sin(\phi_0)}{(x_0)} = 2 + \beta(z_0),$$

we find

$$\frac{1}{\rho_0} - \left[\frac{1}{\rho_0}\right] = -\sin(\phi_0)\left\{\frac{1}{x_0} - \frac{1}{(x_0)}\right\} - \frac{251}{720}\,\omega\eta\,\frac{\cos(\phi_0)}{(x_0)}$$
$$- \frac{\sin(\phi_0)}{(x_0)^2}\left\{\left(\frac{251}{720}\,\omega\right)^2 \eta\sin(\phi_0)\right\}$$
$$+ \beta\left(\frac{251}{720}\,\omega\right)^2 \eta\cos(\phi_0),$$

or

$$\frac{1}{\rho_0} - \left[\frac{1}{\rho_0}\right] = -2\,\frac{\sin(\phi_0)}{(x_0)^2}\left\{\left(\frac{251}{720}\,\omega\right)^2 \eta\sin(\phi_0)\right\} - \frac{251}{720}\,\omega\eta\,\frac{\cos(\phi_0)}{(x_0)}$$
$$+ \beta\left(\frac{251}{720}\,\omega\right)^2 \eta\cos(\phi_0);$$

but

$$\frac{1}{\rho_0} - \left(\frac{1}{\rho_0}\right) = \eta, \text{ by supposition.}$$

Hence $\left[\frac{1}{\rho_0}\right] - \left(\frac{1}{\rho_0}\right) = \eta\left\{1 + 2\left(\frac{251}{720}\,\omega\right)^2\left(\frac{\sin(\phi_0)}{(x_0)}\right)^2 + \frac{251}{720}\,\omega\,\frac{\cos(\phi_0)}{(x_0)} - \beta\left(\frac{251}{720}\,\omega\right)^2\cos(\phi_0)\right\}$,

or

$$\epsilon = \eta\left\{1 + 2\left(\frac{251}{720}\,\omega\right)^2\left(\frac{\sin(\phi_0)}{(x_0)}\right)^2 + \frac{251}{720}\,\omega\,\frac{\cos(\phi_0)}{(x_0)} - \beta\left(\frac{251}{720}\,\omega\right)^2\cos(\phi_0)\right\},$$

which gives η.

Whence the values of $\frac{1}{\rho_0}$, ϕ_0, x_0 and z_0 become known.

If terms involving differences of the 5th order be included, the coefficient $\frac{251}{720}$ in the above expressions must be replaced everywhere by $\frac{95}{288}$.

As in the former case, the following slight modification of the above process will be found convenient in practice.

Suppose $\left(\frac{1}{\rho_0}\right)$ the assumed value of $\frac{1}{\rho_0}$ to be increased by 100 units of the last place of decimals employed, then while calculating the values of $(d\phi_0)$, (ϕ_0), (dx_0), (dz_0), (x_0), (z_0) and the consequent value of $\left[\frac{1}{\rho_0}\right]$, note at the side of the work the changes which would be severally caused in each of these quantities by such an augmentation of $\left(\frac{1}{\rho_0}\right)$.

As before, it may be remarked that if we stop at the terms involving Δ^4 the changes in (ϕ_0), (x_0) and (z_0) will be $\frac{251}{720}$ times the changes in $(d\phi_0)$, (dx_0) and (dz_0) respectively, and that $\frac{251}{720}$ may be conveniently put under the form

$$\frac{1}{3}\left[1+\frac{1}{20}\left(1-\frac{1}{12}\right)\right].$$

If we also include the terms involving Δ^5, the coefficient $\frac{251}{720}$ must be replaced by $\frac{95}{288}$, or $\frac{1}{3}\left(1-\frac{1}{96}\right)$.

Now suppose that an increase of 100 units in $\left(\frac{1}{\rho_0}\right)$ causes a diminution of μ units in $\left[\frac{1}{\rho_0}\right]$, and that the excess of $\left[\frac{1}{\rho_0}\right]$ above $\left(\frac{1}{\rho_0}\right)$ is λ of the same units, then the correction to be applied to the assumed value $\left(\frac{1}{\rho_0}\right)$ will be

$$\frac{100\,\lambda}{100+\mu} \text{ such units,}$$

and the correction to the value of $\left[\frac{1}{\rho_0}\right]$ will be

$$-\frac{\lambda\mu}{100+\mu} \text{ such units,}$$

and the proportionate changes required in the values of $(d\phi_0)$, (ϕ_0), (dx_0), (dz_0), (x_0) and (z_0) will be at once found.

A numerical example of the method, when s is taken as the independent variable, is given hereafter.

As before, we may, if it is found convenient, increase or diminish the interval between the successive values of s.

It may be remarked, as before, that when, by means of the appropriate series, we have found the values of $\frac{1}{\rho}$ for a sufficient number of small values of s, we can form the corresponding value of $d\phi$, and thence derive the corresponding values of ϕ by integration, and again by means of these we can find the corresponding values of dx and dz, and thence derive by integration the corresponding values of x and z without employing the series for those quantities, unless we choose to do so as a means of verification.

In the foregoing investigation, ϕ is supposed to be expressed in the circular measure, but in order to find $\cos\phi$ and $\sin\phi$ from the tables, ϕ must be converted into degrees, minutes and seconds. This can be readily done by means of special tables for the purpose.

Or we may, if we choose, express $d\phi$ at once in seconds by multiplying by 206264·8, the number of seconds in the unit of circular measure, and thence find the number of seconds in ϕ by integration, without passing through the circular measure. Thus if ϕ'' denote the number of seconds in ϕ, and if γ denote the number 206264·8, we shall have

$$d\phi'' = \frac{1}{\rho}\gamma\, ds = \frac{1}{\rho}\gamma\omega,$$

where $\log\gamma = 5{\cdot}3144251$.

In conclusion, it may be worth while to say a few words in order to point out the distinction between the method of integration above explained and that which is commonly known under the name of "Integration by Quadratures." In this latter method, we have to find y from the equation

$$\frac{dy}{dt} = q = f(t),$$

where $f(t)$ is a known function of t.

If we regard q as the ordinate of any point of a curve corresponding to the abscissa t, then y will be the area included between the curve, the axis of t, the ordinate q, and some fixed ordinate.

In this case the values of q can be found, à priori, for any given values of t, whereas in the more general case already treated of, where q is a function of y as well as of t, the unknown quantities y and q must be found simultaneously, and therefore we can only proceed step by step.

As the simpler case is included in the more general one, we may, of course, still employ the same formula of integration that we have already obtained, but it will be more advantageous to use a slightly different one.

If we denote the successive values of q by

$$\ldots q_{n-2},\ q_{n-1},\ q_n,\ \&c.,$$

and if the corresponding values of y be denoted by

$$\ldots y_{n-2},\ y_{n-1},\ y_n,\ \&c.,$$

and if, in a notation similar to that already employed, the successive differences of the quantities q be represented as in the following scheme:

$$\begin{array}{cccccccccccc}
\ldots & & & & & & & & & & & \\
 & \ldots & & \ldots & & \ldots & & \ldots & & \ldots & & \\
 & & \ldots & & \ldots & & \ldots & & \ldots & & & \\
q_{n-1} & & & & & & & & & & & \\
 & \Delta q_n & & \Delta^3 q_{n+1} & & \Delta^5 q_{n+2} & & \Delta^7 q_{n+3} & & \Delta^9 q_{n+4} & & \\
q_n & & \Delta^2 q_{n+1} & & \Delta^4 q_{n+2} & & \Delta^6 q_{n+3} & & \Delta^8 q_{n+4} & & & \&c., \\
 & \Delta q_{n+1} & & \Delta^3 q_{n+2} & & \Delta^5 q_{n+3} & & \Delta^7 q_{n+4} & & \Delta^9 q_{n+5} & & \\
q_{n+1} & & \ldots & & \ldots & & \ldots & & \ldots & & & \\
 & \ldots & & \ldots & & \ldots & & \ldots & & \ldots & & \\
\ldots & & & & & & & & & & &
\end{array}$$

then, as is before proved, we shall have

$$y_n - y_{n-1} = \int q\,dt \text{ between the limits } t = t_0 + (n-1)\,\omega \text{ and } t = t_0 + n\omega$$

$$= \omega \left\{ q_n - \frac{1}{2}\Delta q_n - \frac{1}{12}\Delta^2 q_n - \frac{1}{24}\Delta^3 q_n - \frac{19}{720}\Delta^4 q_n - \frac{3}{160}\Delta^5 q_n - \frac{863}{60480}\Delta^6 q_n \right.$$

$$\left. - \frac{275}{24192}\Delta^7 q_n - \frac{33953}{3628800}\Delta^8 q_n - \frac{8183}{1036800}\Delta^9 q_n - \&c. \right\}.$$

If we transform this series into one which contains only such differences as, in the above scheme, occur in the same horizontal lines as q_n and Δq_n, we shall find that the coefficients of the successive differences ultimately diminish much more rapidly than before.

When the differences of higher orders than the 9th are neglected, it is readily shewn that the above series is equivalent to

$$y_n - y_{n-1} = \omega \left\{ q_n - \frac{1}{2}\Delta q_n - \frac{1}{12}\Delta^2 q_{n+1} + \frac{1}{24}\Delta^3 q_{n+1} + \frac{11}{720}\Delta^4 q_{n+2} - \frac{11}{1440}\Delta^5 q_{n+2} \right.$$

$$\left. - \frac{191}{60480}\Delta^6 q_{n+3} + \frac{191}{120960}\Delta^7 q_{n+3} + \frac{2497}{3628800}\Delta^8 q_{n+4} - \frac{2497}{7257600}\Delta^9 q_{n+4} - \&c. \right\}.$$

Similarly, by repeated applications of this formula, we have

$$y_{n-1} - y_{n-2} = \omega \left\{ q_{n-1} - \frac{1}{2}\Delta q_{n-1} - \frac{1}{12}\Delta^2 q_n + \frac{1}{24}\Delta^3 q_n + \frac{11}{720}\Delta^4 q_{n+1} - \frac{11}{1440}\Delta^5 q_{n+1} \right.$$

$$\left. - \frac{191}{60480}\Delta^6 q_{n+2} + \frac{191}{120960}\Delta^7 q_{n+2} + \frac{2497}{3628800}\Delta^8 q_{n+3} - \frac{2497}{7257600}\Delta^9 q_{n+3} - \&c. \right\},$$

&c., &c.

$$y_{m+1} - y_m = \omega \left\{ q_{m+1} - \frac{1}{2}\Delta q_{m+1} - \frac{1}{12}\Delta^2 q_{m+2} + \frac{1}{24}\Delta^3 q_{m+2} + \frac{11}{720}\Delta^4 q_{m+3} - \frac{11}{1440}\Delta^5 q_{m+3} \right.$$

$$\left. - \frac{191}{60480}\Delta^6 q_{m+4} + \frac{191}{120960}\Delta^7 q_{m+4} + \frac{2497}{3628800}\Delta^8 q_{m+5} - \frac{2497}{7257600}\Delta^9 q_{m+5} - \&c. \right\}.$$

Adding all these equations, and observing that

$$\begin{aligned}
\Delta q_n + \Delta q_{n-1} + \&c. + \Delta q_{m+1} &= q_n - q_m,\\
\Delta^2 q_{n+1} + \Delta^2 q_n + \&c. + \Delta^2 q_{m+2} &= \Delta q_{n+1} - \Delta q_{m+1},\\
\Delta^3 q_{n+1} + \Delta^3 q_n + \&c. + \Delta^3 q_{m+2} &= \Delta^2 q_{n+1} - \Delta^2 q_{m+1},\\
\&c. &= \&c.\\
\Delta^9 q_{n+4} + \Delta^9{}_{n+3} + \&c. + \Delta^9 q_{m+5} &= \Delta^8 q_{n+4} - \Delta^8 q_{m+4},
\end{aligned}$$

we obtain

$$y_n - y_m = \int q\,dt \text{ between the limits } t = t_0 + m\omega \text{ and } t = t_0 + n\omega,$$

$$= \omega \left\{ q_{m+1} + q_{m+2} + \&c. + q_{n-1} + q_n - \frac{1}{2}(q_n - q_m) \right.$$

$$- \frac{1}{12}(\Delta q_{n+1} - \Delta q_{m+1}) + \frac{1}{24}(\Delta^2 q_{n+1} - \Delta^2 q_{m+1}) + \frac{11}{720}(\Delta^3 q_{n+2} - \Delta^3 q_{m+2})$$

$$- \frac{11}{1440} (\Delta^4 q_{n+2} - \Delta^4 q_{m+2}) - \frac{191}{60480} (\Delta^5 q_{n+3} - \Delta^5 q_{m+3}) + \frac{191}{120960} (\Delta^6 q_{n+3} - \Delta^6 q_{m+3})$$
$$+ \frac{2497}{3628800} (\Delta^7 q_{n+4} - \Delta^7 q_{m+4}) - \frac{2497}{7257600} (\Delta^8 q_{n+4} - \Delta^8 q_{m+4}) - \&c.\Big\},$$

in the first line of which expression

$$q_{m+1} + q_{m+2} + \&c. + q_n - \tfrac{1}{2}(q_n - q_m)$$

may be replaced by

$$\tfrac{1}{2}(q_m + q_n) + q_{m+1} + q_{m+2} + \&c. + q_{n-1}.$$

Also, by substituting for the differences of odd orders in the series for $y_n - y_{n-1}$, viz. by putting

$$\begin{aligned} \Delta q_n &= \quad q_n \; - q_{n-1}, \\ \Delta^3 q_{n+1} &= \Delta^2 q_{n+1} - \Delta^2 q_n, \\ \&c. & \quad \&c., \end{aligned}$$

we obtain

$$y_n - y_{n-1} = \omega \left\{ \frac{1}{2}(q_n + q_{n-1}) - \frac{1}{24}(\Delta^2 q_{n+1} + \Delta^2 q_n) + \frac{11}{1440}(\Delta^4 q_{n+2} + \Delta^4 q_{n+1}) \right.$$
$$\left. - \frac{191}{120960}(\Delta^6 q_{n+3} + \Delta^6 q_{n+2}) + \frac{2497}{7257600}(\Delta^8 q_{n+4} + \Delta^8 q_{n+3}) - \&c. \right\},$$

and similarly, by substituting for the differences of even orders in the series for $y_n - y_m$, viz. by putting

$$\begin{aligned} \Delta^2 q_{n+1} &= \Delta q_{n+1} - \Delta q_n, & \Delta^2 q_{m+1} &= \Delta q_{m+1} - \Delta q_m, \\ \Delta^4 q_{n+2} &= \Delta^3 q_{n+2} - \Delta^3 q_{n+1}, & \Delta^4 q_{m+2} &= \Delta^3 q_{m+2} - \Delta^3 q_{m+1}, \\ &\&c. & &\&c., \end{aligned}$$

we obtain

$$y_n - y_m = \omega \left\{ \frac{1}{2}(q_m + q_n) + q_{m+1} + q_{m+2} + \&c. + q_{n-1} \right.$$
$$- \frac{1}{24}(\Delta q_n + \Delta q_{n+1}) + \frac{1}{24}(\Delta q_m + \Delta q_{m+1})$$
$$+ \frac{11}{1440}(\Delta^3 q_{n+1} + \Delta^3 q_{n+2}) - \frac{11}{1440}(\Delta^3 q_{m+1} + \Delta^3 q_{m+2})$$
$$- \frac{191}{120960}(\Delta^5 q_{n+2} + \Delta^5 q_{n+3}) + \frac{191}{120960}(\Delta^5 q_{m+2} + \Delta^5 q_{m+3})$$
$$\left. + \frac{2497}{7257600}(\Delta^7 q_{n+3} + \Delta^7 q_{n+4}) - \frac{2497}{7257600}(\Delta^7 q_{m+3} + \Delta^7 q_{m+4}) - \&c. \right\}.$$

When, by means of the method before explained, we have found a series of successive values of q, viz.

$$q_m, \; q_{m+1}, \; \&c., \; q_{n-1}, \; q_n,$$

together with the differences of odd orders which are immediately contiguous to the horizontal lines through q_m and q_n, we may advantageously employ the formula just obtained in verification of the value of $y_n - y_m$ previously found.

Weddle's approximate formula for the area of a curve which is divided into 6 portions by 7 given equidistant ordinates*, viz.

$$y_6 - y_0 = \frac{3\omega}{10}\{q_0 + q_2 + q_4 + q_6 + 5(q_1 + q_5) + 6q_3\},$$

has likewise been found to afford a convenient means of verification.

NOTE. In the reference made at the top of p. 31 to Bertrand's paper, the page referred to should be 208 instead of 185, the latter being the page at which the paper begins.

EXAMPLE OF THE METHOD, WHEN ϕ IS TAKEN AS THE INDEPENDENT VARIABLE.

Suppose that $\beta = 6$, and that the values of x and z, and also that of ρ, have been already calculated for values of ϕ at intervals of $2\frac{1}{2}^\circ$ from 0° to $32\frac{1}{2}^\circ$, and that we wish to find the values of the same quantities for $\phi = 35^\circ$.

Here

$$\begin{aligned} \omega &= \text{the circular measure of } 2\tfrac{1}{2}^\circ, \\ &= 0{\cdot}04363323, \\ \log \omega &= 8{\cdot}6398174. \end{aligned}$$

In the first place calculate a table giving the logarithms of $\omega \cos \phi$, $\omega \sin \phi$ and $\sin \phi$ for values of ϕ at intervals of $2\frac{1}{2}^\circ$. Thus for $\phi = 35^\circ$ the calculation will be

ω	8·6398174		8·6398174
$\cos \phi$	9·9133645	$\sin \phi$	9·7585913
	8·5531819		8·3984087

The following is a portion of the table.

TABLE A.

ϕ	$\log(\omega \cos \phi)$	$\log(\omega \sin \phi)$	$\log(\sin \phi)$
...	....	....	....
30°	8·5773480	8·3387874	9·6989700
$32\frac{1}{2}$	8·5658466	8·3700339	9·7302165
35	8·5531819	8·3984087	9·7585913
$37\frac{1}{2}$	8·5392841	8·4242645	9·7844471
40	8·5240714	8·4478849	9·8080675
...	....	....	....

* Boole's *Finite Differences*, p. 39.

Next, collect in a table the values of $\log\rho$ for the successive values of ϕ up to $\phi=32\frac{1}{2}^\circ$, and find their differences to the 4th order, thus

TABLE I.

ϕ	$\log\rho$	Δ	Δ^2	Δ^3	Δ^4
...	......				
$22\frac{1}{2}^\circ$	9·8858160				
		− 198034		+ 2502	
25	9·8660126		− 1245		− 633
		− 199279		+ 1869	
$27\frac{1}{2}$	9·8460847		+ 624		− 420
		− 198655		+ 1449	
30	9·8262192		+ 2073		− (400)
		− 196582		(+ 1049)	
$32\frac{1}{2}$	9·8065610		(+ 3122)		
		(− 193460)			
35	(9·7872150)				

In order to find an approximate value of $\log\rho$ for $\phi=35^\circ$, assume a value for the 4th difference immediately following those already found in the table, and by means of this form successively the approximate values of the 3rd, 2nd and 1st differences and of the $\log\rho$ for $\phi=35^\circ$. These values are placed in parentheses to indicate that they are only approximate.

In the above case, we have assumed the next 4th difference to be − 400.

Collect in a table the values of $dx=\rho\omega\cos\phi$ for the successive values of ϕ up to $\phi=32\frac{1}{2}^\circ$, forming them by means of the known values of $\log\rho$ and the logarithms in the 1st column of Table A.

Add to this table the approximate value of dx for $\phi=35^\circ$, formed by means of the approximate value of $\log\rho$ just found, and find the differences of these quantities to the 4th order, thus

TABLE II.

ϕ	dx	Δdx	$\Delta^2 dx$	$\Delta^3 dx$	$\Delta^4 dx$
...	......				
		......			
$22\frac{1}{2}^\circ$	·03099194		+ 2802		− 968
		− 194464		+ 2314	
25	·02904730		+ 5116		− 861
		− 189348		+ 1453	
$27\frac{1}{2}$	·02715382		+ 6569		− 585
		− 182779		+ 868	
30	·02532603		+ 7437		(− 462)
		− 175342		(+ 406)	
$32\frac{1}{2}$	·02357261		(+ 7843)		
		(− 167499)			
35	(·02189762)				

For $\phi = 35^\circ$ the calculation will be

$$\begin{array}{rll} \log(\rho) & 9{\cdot}7872150 & +100 \\ \log(\omega\cos\phi) & \underline{8{\cdot}5531819} & \\ \log & \underline{8{\cdot}3403969} & \\ (dx) & {\cdot}02189762 & +50{\cdot}3 \end{array}$$

The change of (dx) in units of the last decimal place, which would be produced by an increase in $\log(\rho)$ of 100 units in the last decimal, is placed at the side.

Similarly, collect in a table the values of $dz = \rho\omega\sin\phi$ for the same values of ϕ, forming them by means of the logarithms in the 2nd column of Table A. Add to this table the approximate value of dz for $\phi = 35^\circ$, and find the differences, as before, to the 4th order, thus

TABLE III.

ϕ	dz	Δdz	$\Delta^2 dz$	$\Delta^3 dz$	$\Delta^4 dz$
...	......		......		
		......		......	
$22\frac{1}{2}^\circ$	·01283728		− 13145		+ 79
		+ 70770		+ 1416	
25	·01354498		− 11729		− 68
		+ 59041		+ 1348	
$27\frac{1}{2}$	·01413539		− 10381		− 86
		+ 48660		+ 1262	
30	·01462199		− 9119		(− 136)
		+ 39541		(+ 1126)	
$32\frac{1}{2}$	·01501740		(− 7993)		
		(+ 31548)			
35	(·01533288)				

For $\phi = 35^\circ$ the calculation will be

$$\begin{array}{rll} \log(\rho) & 9{\cdot}7872150 & +100 \\ \log(\omega\sin\phi) & \underline{8{\cdot}3984087} & \\ \log & \underline{8{\cdot}1856237} & \\ (dz) & {\cdot}01533288 & +35{\cdot}3 \end{array}$$

the change of (dz) for an increase of 100 units in $\log(\rho)$ is placed at the side.

Collect in two other tables the successive values of x and z which have been already computed, and form the differences of these quantities to the 4th or 5th orders, by which means any error of consequence that may have crept into the work will at once become apparent.

From the values of (dx), (dz) and their differences, and from the known values of x and z for $\phi = 32\frac{1}{2}°$, find the approximate values of x and z for $\phi = 35°$, thus

For $\phi = 32\frac{1}{2}°$, x	·45780303	
$\phi = 35°$, (dx)	·02189762	
$-\frac{1}{2}\,\Delta\,(dx)$	83749,5	50,3
$-\frac{1}{12}\,\Delta^2(dx)$	$-$ 653,6	2,5
$-\frac{1}{24}\,\Delta^3(dx)$	$-$ 16,9	$-$,2
$-\frac{19}{720}\,\Delta^4(dx)$	+ 12,2	3) 52,6
$\phi = 35°$, (x)	·48053156	17,5

For $\phi = 32\frac{1}{2}°$, z	·12246288	
$\phi = 35°$, (dz)	·01533288	
$-\frac{1}{2}\,\Delta\,(dz)$	$-$ 15774	35,3
$-\frac{1}{12}\,\Delta^2(dz)$	666,1	1,8
$-\frac{1}{24}\,\Delta^3(dz)$	$-$ 46,9	$-$,2
$-\frac{19}{720}\,\Delta^4(dz)$	3,6	3) 36,9
(z)	·13764424,8	12,3
	6	6
$\beta\,(z)$	·82586549	74

In order to prevent an accumulation of small errors, the quantities involving Δ, Δ^2, Δ^3 and Δ^4 are carried to one place of decimals beyond those which are ultimately retained.

At the side are calculated the changes of (x) and (z), in units of the 8th place of decimals, which would be required if $\log(\rho)$ were increased by 100 units of the 7th place of decimals.

These changes are found by multiplying the corresponding changes of (dx) and (dz) already found by

$$\frac{251}{720} = \frac{1}{3}\left\{1 + \frac{1}{20}\left(1 - \frac{1}{12}\right)\right\}.$$

Next, from (x) and (z) find $\frac{1}{[\rho]}$ and $\log[\rho]$ by the formula

$$\frac{1}{[\rho]} = 2 + \beta\,(z) - \frac{\sin\phi}{(x)}, \quad \beta \text{ being here} = 6.$$

Table (A), log sin 35°	9·7585913	
$\log(x)$	9·6817219	1,5
log	0·0768694	$-$ 1,5
N°.	1·1936290	$-$ 4,

$2 + 6\,(z) =$	2·8258655	7,4
subtract	1·1936290	$-$ 4,
$\frac{1}{[\rho]}$	1·6322365	11,4
$\log\frac{1}{[\rho]}$	0·2127831	3,0
or $\log[\rho]$	9·7872169	$-$ 3,0
$\log(\rho)$	9·7872150	
Difference	19	

The numbers placed at the side are the changes which would be caused by the increase in $\log(\rho)$ before mentioned.

It should be remarked that in this last calculation the quantity $2+6(z)$ is only carried to 7 places of decimals, whereas in the above calculation $6(z)$ was given to 8 places.

Consequently the change of $2+6(z)$ or that of $6(z)$ before found must be divided by 10, in order to reduce it to units of the 7th decimal.

We see that the value thus found for $\log[\rho]$ exceeds the assumed value of $\log(\rho)$ by 19 units in the 7th decimal place.

Hence the correction to be applied to $\log(\rho)$ will be

$$19 \times \frac{100}{103} = 18{,}4 \text{ such units,}$$

and the corrected value of $\log \rho$ for $\phi = 35^\circ$ will be

$$9{\cdot}7872168,$$

which may now be added to the numbers in Table I.

Similarly, the corrections to be applied to the values of (dx) and (dz) will be

$$50{,}3 \times {\cdot}184 = 9{,}3 \quad \text{and} \quad 35{,}3 \times {\cdot}184 = 6{,}5$$

respectively, in units of the 8th decimal, so that the corrected values will be

$$dx = {\cdot}02189771 \quad \text{and} \quad dz = {\cdot}01533294,$$

which may be added to the numbers in Tables II. and III.

The successive differences of $\log(\rho)$ will of course require the same correction as $\log(\rho)$ itself, and similarly for the differences of (dx) and (dz).

Also, the corrections to be applied to the values of (x) and (z) will be

$$17{,}5 \times {\cdot}184 = 3{,}2 \quad \text{and} \quad 12{,}3 \times {\cdot}184 = 2{,}3$$

respectively, in units of the 8th decimal, so that the corrected values will be

$$x = {\cdot}48053159 \quad \text{and} \quad z = {\cdot}13764427,$$

which may be added to the Tables of the collected values of x and z respectively.

The provisional values of $\log(\rho)$, (dx) and (dz) and of their respective differences, which in the foregoing example have been inclosed in parentheses, may in the actual work be merely written in pencil, so that, when they have served their purpose, they may be easily effaced, and then replaced by the corrected values written in ink.

The Volume V of the portion of the drop corresponding to $\phi = 35^\circ$, is at once found by the formula

$$V = \frac{\pi}{\beta} x^2 \left(\frac{1}{\rho} - \frac{\sin\phi}{x}\right)$$

thus,

$\frac{1}{\rho}$	1·6322367
$\frac{\sin\phi}{x}$	1·1936290
	0·4386077
log	9·6420762
x^2 log	9·3634438
π log	0·4971499
	9·5026699
β log	0·7781513
log	8·7245186
V	·05302963

In this as well as in the former part of the work of this example, b is supposed to be unity, so that in the general case the quantities above denoted by ρ, x, z and V will be replaced by $\frac{\rho}{b}$, $\frac{x}{b}$, $\frac{z}{b}$ and $\frac{V}{b^3}$ respectively.

The following shews the application of the formula of correction

$$\eta = \frac{\epsilon}{1 + \frac{251}{720}\omega(\rho_0)^2 \sin\phi_0 \left[\frac{\cos\phi_0}{(x_0)^2} + \beta\right]}$$

to this example.

		$\cos\phi_0$	9·91336
$\log x_0$	9·68172	$(x_0)^2$	9·36344
			0·54992
		N°.	3·5475
		β	6·
251	2·39967		9·5475
720	2·85733	log	0·97989
	9·54234	$(\rho_0)^2$	9·57443
		$\sin\phi_0$	9·75859
		ω	8·63982
		$\frac{251}{720}$	9·54234
			8·49507
			·031266
			1·
			1·031266

Hence $$\eta = \frac{\epsilon}{1{\cdot}031} \text{ nearly,}$$

which agrees very well with the result found by the other method.

EXAMPLE OF THE METHOD, WHEN s IS TAKEN AS THE INDEPENDENT VARIABLE.

Suppose that it is required to calculate the theoretical form of a pendent drop of fluid, where $\beta = -0{\cdot}5$.

As in the former example, we will suppose that $b = 1$.

First, putting $\beta = -0{\cdot}5$ in the general formula for $\frac{1}{\rho}$, we obtain for points near the origin

$$\frac{1}{\rho} = 1 - 0{\cdot}1875\, s^2 + 0{\cdot}01692{,}7083\, s^4 - 0{\cdot}00248{,}2096\, s^6$$
$$+ 0{\cdot}00028{,}3075\, s^8 - 0{\cdot}00003{,}3537\, s^{10} + \&c.,$$

which series is sufficient to give $\frac{1}{\rho}$ to 9 or 10 places of decimals, provided s do not exceed 0·4.

The value of $\frac{1}{\rho}$ is the same for corresponding positive and negative values of s, so that if we put $s = 0$, $s = \pm 0{\cdot}1$, $s = \pm 0{\cdot}2$, $s = \pm 0{\cdot}3$ and $s = \pm 0{\cdot}4$ in succession, we may obtain

s	$\frac{1}{\rho}$
0	1·
± 0·1	0·99812,67
± 0·2	0·99252,69
± 0·3	0·98326,03
± 0·4	0·97042,33

Also
$$\phi = \int \frac{1}{\rho}\, ds = s - 0{\cdot}0625\, s^3 + 0{\cdot}00338{,}54166\, s^5 - 0{\cdot}00035{,}4585\, s^7$$
$$+ 0{\cdot}00003{,}1453\, s^9 - 0{\cdot}00000{,}3049\, s^{11} + \&c.$$

Similarly, putting $\beta = -0{\cdot}5$ in the series for x and z respectively, we obtain

$$x = s - 0{\cdot}166\dot{6}\, s^3 + 0{\cdot}02083{,}\dot{3}\, s^5 - 0{\cdot}00244{,}9157\, s^7 + 0{\cdot}00029{,}4734\, s^9$$
$$- 0{\cdot}00003{,}5199\, s^{11} + \&c.,$$

$$z = 0{\cdot}5\, s^2 - 0{\cdot}05729{,}1\dot{6}\, s^4 + 0{\cdot}00716{,}14583\, s^6 - 0{\cdot}00085{,}03747\, s^8$$
$$+ 0{\cdot}00010{,}17165\, s^{10} - 0{\cdot}00001{,}21847\, s^{12} + \&c.$$

From which we may find

s	ϕ (in circ. measure).	ϕ (in deg. &c.)	x	z
0	0·0	0° 0′ 0″	0·0	0·0
± 0·1	± 0·09993,753	± 5 43 33·596	± 0·09983,354	0·00499,428
± 0·2	± 0·19950,108	± 11 25 50·051	± 0·19867,330	0·01990,879
± 0·3	± 0·29832,065	± 17 5 33·051	± 0·29555,009	0·04454,110
± 0·4	± 0·39603,409	± 22 41 27·896	± 0·38954,273	0·07856,212

Now, let us suppose that the values of ρ, ϕ, x and z have been already calculated for $s=0{\cdot}1$, $s=0{\cdot}2$ and $s=0{\cdot}3$, and that we wish to find the values of the same quantities for $s=0{\cdot}4$ by the foregoing method of integration.

Here we have $$\omega=0{\cdot}1.$$

From the given values of $\dfrac{1}{\rho}$ we may find the corresponding values of $d\phi=\omega\dfrac{1}{\rho}$, and their successive differences, as shewn in the following Table:

s	$d\phi$	$\Delta d\phi$	$\Delta^2 d\phi$	$\Delta^3 d\phi$	$\Delta^4 d\phi$	$\Delta^5 d\phi$
− 0·3	0·09832,603					
		92,666				
− 0·2	0·09925,269		−36,668			
		55,998		−0,597		
− 0·1	0·09981,267		−37,265		+0,396	
		18,733		−0,201		+,006
0·0	0·1		−37,466		+0,402	
		− 18,733		+0,201		−,006
0·1	0·09981,267		−37,265		+0,396	
		− 55,998		+0,597		(−,018)
0·2	0·09925,269		−36,668		(+0,378)	
		− 92,666		(+0,975)		
0·3	0·09832,603		(−35,693)			
		(−128,359)				
0·4	(0·09704,244)					

If we supply another line of differences by supposing the 6th difference to be constant, we shall obtain the quantities included in parentheses in the above Table, and the corresponding assumed value of $\dfrac{1}{(\rho)}$ for $s=0{\cdot}4$ will be 0·97042,44.

From the values of $(d\phi)$ and its differences, and the known value of ϕ for $s=0{\cdot}3$, find the approximate value of ϕ for $s=0{\cdot}4$, thus

For $s=0{\cdot}3$,	ϕ	·29832,065	
$s=0{\cdot}4$,	$d\phi$	·09704,244	
	$-\frac{1}{2}\Delta\, d\phi$	+ 64,179,5	
	$-\frac{1}{12}\Delta^2 d\phi$	+ 2,974,4	100 in units of 8th decimal place
	$-\frac{1}{24}\Delta^3 d\phi$	− ,040,6	−1
	$-\frac{19}{720}\Delta^4 d\phi$	− ,010	3) 99
	$-\frac{3}{160}\Delta^5 d\phi$	+ ,000,4	
	(ϕ)	·39603,413	33
		or 22° 41′ 27″·903	0″·068

The changes placed at the side correspond to an increase in $\dfrac{1}{(\rho)}$ of 100 units in the 7th decimal place.

As the interval ω is rather large, we have taken into account the terms in Δ^5.

With this value of (ϕ) we calculate the corresponding values of (dx) and (dz), thus

$\log\cos(\phi)$	9·9650125	$-0,6$	$\log\sin(\phi)$	9·5863199	$+3,4$
$\log\omega$	9·		$\log\omega$	9·	
	8·9650125			8·5863199	
(dx)	·09225980	$-1,2$	(dz)	·03857624	$+3,0.$

The small quantities at the side are the changes of the quantities opposite to which they stand, in units of the last decimals respectively employed, which would be caused by an increase of 0″·068 in (ϕ), or by an increase of 100 units in the 7th decimal place of $\dfrac{1}{(\rho)}$.

With these values of (dx) and (dz) and the previously calculated values of dx and dz for $s = 0·1$, $s = 0·2$ and $s = 0·3$, we may form the following Tables:

s	dx	Δdx	$\Delta^2 dx$	$\Delta^3 dx$	$\Delta^4 dx$	$\Delta^5 dx$
− 0·3	·09558,313					
		243,344				
− 0·2	·09801,657		− 94,895			
		148,449		− 3,660		
− 0·1	·09950,106		− 98,555		+ 2,427	
		49,894		− 1,233		+ 39
0·	·1		− 99,788		+ 2,466	
		− 49,894		+ 1,233		− 39
0·1	·09950,106		− 98,555		+ 2,427	
		− 148,449		+ 3,660		(− 181)
0·2	·09801,657		− 94,895		(+ 2,246)	
		− 243,344		(+ 5,906)		
0·3	·09558,313		(− 88,989)			
		(− 332,333)				
0·4	(·09225,980)					

s	dz	Δdz	$\Delta^2 dz$	$\Delta^3 dz$	$\Delta^4 dz$	$\Delta^5 dz$
− 0·3	− ·02939,154					
		957,351				
− 0·2	− ·01981,803		26,740			
		984,091		− 13,119		
− 0·1	− ·00997,712		13,621		− 502	
		997,712		− 13,621		502
− 0·	·0		0		0	
		997,712		− 13,621		502
0·1	+ ·00997,712		− 13,621		+ 502	
		984,091		− 13,119		(476)
0·2	·01981,803		− 26,740		(+ 978)	
		957,351		(− 12,141)		
0·3	·02939,154		(− 38,881)			
		(918,470)				
0·4	(·03857,624)					

Hence we find x and z for $s = 0{\cdot}4$, thus

For $s = 0{\cdot}3$,	$x = {\cdot}29555009$			For $s = 0{\cdot}3$,	$z = {\cdot}04454110$		
For $s = 0{\cdot}4$,	$dx = {\cdot}09225980$			For $s = 0{\cdot}4$,	dz	${\cdot}03857624$	
$-\ \frac{1}{2}\Delta\, dx$	166166,5			$-\ \frac{1}{2}\Delta\, dz$		-459235	
$-\ \frac{1}{12}\Delta^2 dx$	7415,7			$-\ \frac{1}{12}\Delta^2 dz$		3240,1	
$-\ \frac{1}{24}\Delta^3 dx$	$-246,1$			$-\ \frac{1}{24}\Delta^3 dz$		505,9	
$-\frac{19}{720}\Delta^4 dx$	$-\ 59,3$			$-\frac{19}{720}\Delta^4 dz$		$-25,8$	
$-\frac{3}{160}\Delta^5 dx$	3,4			$-\ \frac{3}{160}\Delta^5 dz$		$-\ 8,9$	
(x)	${\cdot}38954269$	$-0,4$		(z)	2)	${\cdot}07856210$	$+1,0$
				$-\beta z$		${\cdot}03928105$	0,5

The changes in (x) and (z) are found by multiplying those in (dx) and (dz) respectively, by $\frac{95}{288}$ or $\frac{1}{3}$ nearly.

Next, with these values of (ϕ), (x) and (z), find $\frac{1}{[\rho]}$ by the formula

$$\frac{1}{[\rho]} = 2 + \beta(z) - \frac{\sin(\phi)}{(x)}, \quad \beta \text{ being here } = -0{\cdot}5.$$

$\log\sin(\phi)$	$9{\cdot}5863199$	$+3,4$		$2+\beta(z)$	$= 1{\cdot}9607189,5$	$-0,05$
$\log(x)$	$9{\cdot}5905551$	0,0		subtract	${\cdot}9902955$	7,7
	$9{\cdot}9957648$	3,4		$\frac{1}{[\rho]}$	$0{\cdot}9704234,5$	$-7,75$
N°.	${\cdot}9902955$	7,7				
				but $\frac{1}{(\rho)}$	$= 0{\cdot}9704244$	
				$\therefore$ difference	$=$ 9,5	

It will be seen that in the above we have expressed the change of $2 + \beta(z)$ in units of the 7th decimal place instead of the 8th decimal as before, so that the number found before has been divided by 10.

The value thus found for $\frac{1}{[\rho]}$ is less than the assumed value of $\frac{1}{(\rho)}$ by 9,5 units in the 7th decimal place.

Hence the correction to be applied to $\frac{1}{(\rho)}$ will be

$$-9,5\,\frac{100}{107,75} = -9 \text{ such units,}$$

and the corrected value of $\frac{1}{\rho}$ will be $0{\cdot}9704235$.

Similarly, the correction to be applied to the value of (ϕ) will be

$$-{\cdot}09 \times {''}{\cdot}068 = -{''}{\cdot}006,$$

so that the corrected value of ϕ will be $22^\circ\,41'\,27''{\cdot}897$.

Also the corrections to be applied to the values of (x) and (z) respectively, will be

$$-\cdot 09 \times -0{,}4 \text{ and } -\cdot 09 \times 1{,}0$$

in units of the 8th decimal place, both of which are insensible.

These results agree very well with the more accurate ones found before by the use of series.

The Volume V is found by the formula

$$V = \frac{\pi}{(-\beta)} x^2 \left(\frac{\sin\phi}{x} - \frac{1}{\rho}\right)$$

thus,

$$\begin{array}{rl}
\dfrac{\sin\phi}{x} & 0\cdot 9902954 \\
\dfrac{1}{\rho} & 0\cdot 9704235 \\ \hline
 & 0\cdot 0198719 \\ \hline
\log & 8\cdot 2982394 \\
x^2 \log & 9\cdot 1811102 \\
\pi & 0\cdot 4971499 \\ \hline
 & 7\cdot 9764995 \\
(-\beta)\log & 9\cdot 6989700 \\ \hline
\log & 8\cdot 2775295 \\ \hline
V & \cdot 0189465
\end{array}$$

Application of the formula of correction

$$\epsilon = \eta \left\{1 + 2\left(\frac{95}{288}\omega\right)^2 \left(\frac{\sin(\phi_0)}{(x_0)}\right)^2 + \frac{95}{288}\omega \cdot \frac{\cos(\phi_0)}{(x_0)} - \beta\left(\frac{95}{288}\omega\right)^2 \cos(\phi_0)\right\}$$

to this example.

$$\begin{array}{rl}
\frac{95}{288}\log & 9\cdot 51833 \\
\omega & 9\cdot \\ \hline
 & 8\cdot 51833 \\
\cos(\phi_0) & 9\cdot 96501 \\ \hline
 & 8\cdot 48334 \\
(x) & 9\cdot 59055 \\ \hline
 & 8\cdot 89279 \\ \hline
 & \cdot 078125 \\
 & \cdot 002134 \\
 & \cdot 000502 \\ \hline
 & \cdot 080761
\end{array}
\qquad
\begin{array}{rl}
\dfrac{\sin(\phi_0)}{(x_0)}\log & 9\cdot 99576 \\
 & 8\cdot 51833 \\ \hline
 & 8\cdot 51409 \\
 & 2 \\ \hline
 & 7\cdot 02818 \\
2\log & 0\cdot 30103 \\ \hline
 & 7\cdot 32921
\end{array}
\qquad
\begin{array}{rl}
(\frac{95}{288}\omega)^2\log & 7\cdot 03666 \\
\cos(\phi_0) & 9\cdot 96501 \\
-\beta & 9\cdot 69897 \\ \hline
 & 6\cdot 70064
\end{array}$$

Hence

$$\eta = \frac{\epsilon}{1\cdot 081},$$

which very nearly agrees with the result found before by the other method.

TABLES

FOR FACILITATING THE CALCULATION OF

$$\frac{19}{720}\Delta^4, \quad \frac{3}{160}\Delta^5, \quad \&c.$$

Table shewing the value of Δ^4 which corresponds to each unit in the value of

$$\frac{19}{720}\Delta^4 = \frac{1}{37{\cdot}8947}\Delta^4.$$

	0	1	2	3	4	5	6	7	8	9	
0	0	38	76	114	152	189	227	265	303	341	0
1	379	417	455	493	531	568	606	644	682	720	1
2	758	796	834	872	909	947	985	1023	1061	1099	2
3	1137	1175	1213	1251	1288	1326	1364	1402	1440	1478	3
4	1516	1554	1592	1629	1667	1705	1743	1781	1819	1857	4
5	1895	1933	1971	2008	2046	2084	2122	2160	2198	2236	5
6	2274	2312	2349	2387	2425	2463	2501	2539	2577	2615	6
7	2653	2691	2728	2766	2804	2842	2880	2918	2956	2994	7
8	3032	3069	3107	3145	3183	3221	3259	3297	3335	3373	8
9	3411	3448	3486	3524	3562	3600	3638	3676	3714	3752	9
10	3789	3827	3865	3903	3941	3979	4017	4055	4093	4131	10
11	4168	4206	4244	4282	4320	4358	4396	4434	4472	4509	11
12	4547	4585	4623	4661	4699	4737	4775	4813	4851	4888	12
13	4926	4964	5002	5040	5078	5116	5154	5192	5229	5267	13
14	5305	5343	5381	5419	5457	5495	5533	5571	5608	5646	14
15	5684	5722	5760	5798	5836	5874	5912	5949	5987	6025	15
16	6063	6101	6139	6177	6215	6253	6291	6328	6366	6404	16
17	6442	6480	6518	6556	6594	6632	6669	6707	6745	6783	17
18	6821	6859	6897	6935	6973	7011	7048	7086	7124	7162	18
19	7200	7238	7276	7314	7352	7389	7427	7465	7503	7541	19
20	7579	7617	7655	7693	7731	7768	7806	7844	7882	7920	20
21	7958	7996	8034	8072	8109	8147	8185	8223	8261	8299	21
22	8337	8375	8413	8451	8488	8526	8564	8602	8640	8678	22
23	8716	8754	8792	8829	8867	8905	8943	8981	9019	9057	23
24	9095	9133	9171	9208	9246	9284	9322	9360	9398	9436	24
25	9474	9512	9549	9587	9625	9663	9701	9739	9777	9815	25
26	9853	9891	9928	9966	10004	10042	10080	10118	10156	10194	26
27	10232	10269	10307	10345	10383	10421	10459	10497	10535	10573	27

The units of $\frac{19}{720}\Delta^4$ are placed at the top of the Table, and the tens at the side.

Table shewing the value of Δ^5 which corresponds to each unit in the value of

$$\frac{3}{160}\Delta^5 = \frac{1}{53 \cdot 3333}\Delta^5.$$

	0	1	2	3	4	5	6	7	8	9	
0	0	53	107	160	213	267	320	373	427	480	0
1	533	587	640	693	747	800	853	907	960	1013	1
2	1067	1120	1173	1227	1280	1333	1387	1440	1493	1547	2
3	1600	1653	1707	1760	1813	1867	1920	1973	2027	2080	3
4	2133	2187	2240	2293	2347	2400	2453	2507	2560	2613	4
5	2667	2720	2773	2827	2880	2933	2987	3040	3093	3147	5
6	3200	3253	3307	3360	3413	3467	3520	3573	3627	3680	6
7	3733	3787	3840	3893	3947	4000	4053	4107	4160	4213	7
8	4267	4320	4373	4427	4480	4533	4587	4640	4693	4747	8
9	4800	4853	4907	4960	5013	5067	5120	5173	5227	5280	9
10	5333	5387	5440	5493	5547	5600	5653	5707	5760	5813	10
11	5867	5920	5973	6027	6080	6133	6187	6240	6293	6347	11
12	6400	6453	6507	6560	6613	6667	6720	6773	6827	6880	12
13	6933	6987	7040	7093	7147	7200	7253	7307	7360	7413	13
14	7467	7520	7573	7627	7680	7733	7787	7840	7893	7947	14
15	8000	8053	8107	8160	8213	8267	8320	8373	8427	8480	15
16	8533	8587	8640	8693	8747	8800	8853	8907	8960	9013	16
17	9067	9120	9173	9227	9280	9333	9387	9440	9493	9547	17
18	9600	9653	9707	9760	9813	9867	9920	9973	10027	10080	18

The units of $\frac{3}{160}\Delta^5$ are placed at the top of the Table, and the tens at the side.

Table shewing the value of Δ^6 which corresponds to each unit in the value of

$$\frac{863}{60480}\Delta^6 = \frac{1}{70{\cdot}0811}\Delta^6.$$

	0	1	2	3	4	5	6	7	8	9	
0	0	70	140	210	280	350	420	491	561	631	0
1	701	771	841	911	981	1051	1121	1191	1261	1332	1
2	1402	1472	1542	1612	1682	1752	1822	1892	1962	2032	2
3	2102	2173	2243	2313	2383	2453	2523	2593	2663	2733	3
4	2803	2873	2943	3013	3084	3154	3224	3294	3364	3434	4
5	3504	3574	3644	3714	3784	3854	3925	3995	4065	4135	5
6	4205	4275	4345	4415	4485	4555	4625	4695	4766	4836	6
7	4906	4976	5046	5116	5186	5256	5326	5396	5466	5536	7
8	5606	5677	5747	5817	5887	5957	6027	6097	6167	6237	8
9	6307	6377	6447	6518	6588	6658	6728	6798	6868	6938	9
10	7008	7078	7148	7218	7288	7359	7429	7499	7569	7639	10
11	7709	7779	7849	7919	7989	8059	8129	8199	8270	8340	11
12	8410	8480	8550	8620	8690	8760	8830	8900	8970	9040	12
13	9111	9181	9251	9321	9391	9461	9531	9601	9671	9741	13
14	9811	9881	9952	10022	10092	10162	10232	10302	10372	10442	14
15	10512	10582	10652	10722	10792	10863	10933	11003	11073	11143	15

The units of $\frac{863}{60480}\Delta^6$ are placed at the top of the Table, and the tens at the side.

Table shewing the value of Δ^7 which corresponds to each unit in the value of

$$\frac{275}{24192}\Delta^7 = \frac{1}{87\cdot9709}\Delta^7.$$

-	0	1	2	3	4	5	6	7	8	9	
0	0	88	176	264	352	440	528	616	704	792	0
1	880	968	1056	1144	1232	1320	1408	1496	1583	1671	1
2	1759	1847	1935	2023	2111	2199	2287	2375	2463	2551	2
3	2639	2727	2815	2903	2991	3079	3167	3255	3343	3431	3
4	3519	3607	3695	3783	3871	3959	4047	4135	4223	4311	4
5	4399	4487	4574	4662	4750	4838	4926	5014	5102	5190	5
6	5278	5366	5454	5542	5630	5718	5806	5894	5982	6070	6
7	6158	6246	6334	6422	6510	6598	6686	6774	6862	6950	7
8	7038	7126	7214	7302	7390	7478	7565	7653	7741	7829	8
9	7917	8005	8093	8181	8269	8357	8445	8533	8621	8709	9
10	8797	8885	8973	9061	9149	9237	9325	9413	9501	9589	10
11	9677	9765	9853	9941	10029	10117	10205	10293	10381	10469	11
12	10557	10644	10732	10820	10908	10996	11084	11172	11260	11348	12

The units of $\frac{275}{24192}\Delta^7$ are placed at the top of the Table, and the tens at the side.

Table shewing the value of Δ^8 which corresponds to each unit in the value of

$$\frac{33953}{3628800}\Delta^8 = \frac{1}{106{\cdot}87715}\Delta^8.$$

	0	1	2	3	4	5	6	7	8	9	
0	0	107	214	321	428	534	641	748	855	962	0
1	1069	1176	1283	1389	1496	1603	1710	1817	1924	2031	1
2	2138	2244	2351	2458	2565	2672	2779	2886	2993	3099	2
3	3206	3313	3420	3527	3634	3741	3848	3954	4061	4168	3
4	4275	4382	4489	4596	4703	4809	4916	5023	5130	5237	4
5	5344	5451	5558	5664	5771	5878	5985	6092	6199	6306	5
6	6413	6520	6626	6733	6840	6947	7054	7161	7268	7375	6
7	7481	7588	7695	7802	7909	8016	8123	8230	8336	8443	7
8	8550	8657	8764	8871	8978	9085	9191	9298	9405	9512	8
9	9619	9726	9833	9940	10046	10153	10260	10367	10474	10581	9

The units of $\frac{33953}{3628800}\Delta^8$ are placed at the top of the Table, and the tens at the side.

Table shewing the value of Δ^9 which corresponds to each unit in the value of

$$\frac{8183}{1036800}\Delta^9 = \frac{1}{126{\cdot}7017}\Delta^9.$$

	0	1	2	3	4	5	6	7	8	9	
0	0	127	253	380	507	634	760	887	1014	1140	0
1	1267	1394	1520	1647	1774	1901	2027	2154	2281	2407	1
2	2534	2661	2787	2914	3041	3168	3294	3421	3548	3674	2
3	3801	3928	4054	4181	4308	4435	4561	4688	4815	4941	3
4	5068	5195	5321	5448	5575	5702	5828	5955	6082	6208	4
5	6335	6462	6588	6715	6842	6969	7095	7222	7349	7475	5
6	7602	7729	7856	7982	8109	8236	8362	8489	8616	8742	6
7	8869	8996	9123	9249	9376	9503	9629	9756	9883	10009	7
8	10136	10263	10390	10516	10643	10770	10896	11023	11150	11276	8
9	11403	11530	11657	11783	11910	12037	12163	12290	12417	12543	9

The units of $\frac{8183}{1036800}\Delta^9$ are placed at the top of the Table, and the tens at the side.

CHAPTER IV.

COMPARISON OF CALCULATED AND MEASURED FORMS OF DROPS.

THE coordinates $\frac{x}{b}$ and $\frac{z}{b}$ for the curves represented by Laplace's differential equation were calculated by the method of Professor Adams for values of ϕ, 5°, 10°, 15° ...... 175°, 180°, and for values of β, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$; $\frac{3}{4}$, 1; $1\frac{1}{2}$, 2, $2\frac{1}{2}$; 3, 4, 5, 6, 7, 8; 10, 12, 14, 16; 20, 24, 28, 32; 40, 48, 56, 64, 72, 80, 88, 96 and 100. For $\beta=1$ the calculations were made by Professor Adams, for $\beta=10$ by Professor W. G. Adams, and for the values of β, $\frac{1}{4}$, $\frac{1}{2}$, 3, 6, 16 and 32 by myself. The calculations for the remaining positive values of β were made by Dr C. Powalky, who was recommended for the work by the late Professor Encke.

Afterwards the values of $\frac{x}{b}$ and $\frac{z}{b}$ corresponding to $\phi=5^\circ$, and for the successive values of β, 0, 8, 16, 24, 32, 40 ... 88 and 96 were arranged in order and differenced. Then the values of $\frac{x}{b}$ and $\frac{z}{b}$ corresponding to the same value of ϕ, and to the values of β, 36, 44 ... 92 were found from the above by interpolation, arranged in order with the values of the same quantities for the values of β, 0, 4, 8, &c. already calculated, and the whole differenced. Next the values of $\frac{x}{b}$ and $\frac{z}{b}$ corresponding to $\phi=5^\circ$ and to the values of β, 18, 22, 26, 30, 34, 38, 42, &c. were found from the above by interpolation, arranged in order with the values of the same quantities for values of β, 0, 2, 4, 6, 8, 10, &c. already found, and the whole differenced. And in the same manner the values of $\frac{x}{b}$ and $\frac{z}{b}$ corresponding to $\phi=5^\circ$ were found by interpolation for the values of β, 9, 11, 13, 15, 17, 19, &c., arranged in order with the values of the same quantities already found for the remaining integral values of β up to 100, and the whole differenced.

Further, the same process was gone through for values of ϕ, 10°, 15°, 20° .... 175° and 180°.

Finally the results were arranged as they have been given in Table II., with ϕ for their argument, and differenced to test the accuracy of the work.

Values of $\frac{V}{b^3}$ were calculated by the formula already given (p. 30) for the same values of β for which $\frac{x}{b}$ and $\frac{z}{b}$ were calculated, and the results have been given in Table III.

Values of $\frac{x}{b}$ and $\frac{z}{b}$ corresponding to values of ϕ, 15°, 30°, 45°, 60°, 90°, 120°, 135°, 140°, 145° and 150° were taken from Table II., and, the intermediate values having been supplied by interpolation, the results have been given in Table V. This Table is useful in calculating the theoretical forms of drops corresponding to given values of β.

Other Tables were formed by interpolation of values of $\frac{x}{b}$, $\frac{z}{b}$ and $\frac{V}{b^3}$ intermediate to those given in the above-mentioned Tables, but they are at once too extensive and yet too incomplete for publication in their present form. In this way Tables of values of $\frac{x}{b}$, $\frac{z}{b}$ and $\frac{V}{b^3}$ were formed corresponding to values of β, 0·0, 0·1, 0·2, 0·3 ...... 49·8, 49·9, 50·0, for values of ϕ, 135°, 136°, 137°, 138° ...... 153°, 154°, 155°, which were extremely useful, as will be explained hereafter, in deducing the capillary constants from the measured forms of drops of mercury. But these Tables would have been still more convenient if they had been formed so as to give $\log\frac{x}{b}$, $\log\frac{z}{b}$ and $\log\frac{V}{b^3}$ instead of $\frac{x}{b}$, $\frac{z}{b}$ and $\frac{V}{b^3}$.

The values of $\frac{x}{b}$ and $\frac{z}{b}$ vary so rapidly for low values of β and high values of ϕ that they could be obtained by interpolation correct only to four places of decimals from $\beta=0$ to $\beta=1·9$. Beyond that they were calculated to five places of decimals, while the values of $\frac{V}{b^3}$ were calculated, unfortunately, to only four places of decimals throughout.

The rectangular coordinates of points in the outlines of drops of mercury were measured by the help of a microscope moveable on vertical and horizontal slides by micrometer screws. The microscope was focused by motion along a third slide parallel to the line of collimation. These three slides were parallel to three rectangular axes, two of which were horizontal.

There was found to be a difficulty in arranging the cross lines of the microscope, so as to obtain a correct outline of the drop of fluid, because it would be difficult to judge when the crossing of the micrometer lines was exactly on the contour of the drop. Much labour was expended on the construction of a position micrometer, where the intersection of the *middle* of one micrometer line with a *side* of the other line was to be the centre, about which they turned. At each observation the micrometer

lines were to be turned so that the above-mentioned *side* of the micrometer line became a tangent to the contour of the drop, while the other line was a normal at that point. But this could not be considered a satisfactory arrangement, because there would be some error of excentricity, and the microscope would be liable to be slightly disturbed by the application of the force necessary to bring the cross lines into their proper position. This contrivance was therefore abandoned in favour of a more simple arrangement suggested to me by the late Professor W. H. Miller. This was to use two pairs of parallel and equidistant spider lines, one pair being horizontal and the other pair vertical, so as to form by their intersection a small square in the centre of the field of view (Fig. 1). This arrangement appears to be quite satisfactory, for it is easy to judge when a small arc of the contour of a drop of fluid passes through the middle of this small central square.

The screws used to measure the vertical and horizontal coordinates of points in the contour of a drop of fluid were originally formed of one piece of metal. Great care was taken to obtain a screw of uniform pitch throughout, of about 53 turns to the inch. When however the screws were mounted, it was found that although each one was tolerably uniform, the two screws differed sensibly in their pitch. By the use of a micrometer ruled to the one-hundredth of an inch, the exact rate of both screws at every point was determined. In this way tables of the value of every turn were calculated for both the vertical and horizontal screws. In the following measurements of the forms of five drops of mercury made in 1863, the original readings of the screws are given as they were entered in the observing book, as well as their values in inches obtained by the help of the above-mentioned Tables.

The coordinates of numerous points in the contours of these five drops of mercury, which vary considerably in size, were measured, because it was desired to find whether the theoretical forms would agree satisfactorily with the true forms of drops of mercury. In Fig. 2, the theoretical forms of these drops are given on a large scale, and an attempt has been made to indicate by a cross the position of some of the points measured.

No. I

Original readings in turns of the screws			Readings converted into inches			The same when the origin of coordinates is at the vertex			Calculated		
z''	x_1''	x_2''	z'	x_1'	x_2'	z	x_1	x_2	ϕ	z	x
			inch	inch	inch	inch	inch	inch		inch	inch
13·282	23·416	30·288	0·2491	0·4410	0·5705	0·0925	0·0650	0·0645	144°·48	0·0925	0·0647
13·300	23·398	30·340	·2495	·4406	·5715	·0921	·0654	·0655	140°·0	·0911	·0666
13·400	23·270	30·440	·2513	·4382	·5733	·0903	·0678	·0673	135°·0	·0892	·0686
13·500	23·170	30·540	·2532	·4363	·5752	·0884	·0697	·0692			
13·600	23·072	30·625	·2551	·4345	·5768	·0865	·0715	·0708			
13·800	22·970	30·772	·2588	·4326	·5796	·0828	·0734	·0736	120°·0	·0822	·0740
14·000	22·879	30·876	·2626	·4308	·5816	·0790	·0752	·0756			
14·200	22·797	30·950	·2663	·4293	·5830	·0753	·0767	·0770			
14·400	22·735	31·003	·2701	·4281	·5840	·0715	·0779	·0780			
14·600	22·680	31·033	·2738	·4271	·5845	·0678	·0789	·0785			
14·960	22·672	31·064	·2806	·4269	·5851	·0610	·0791	·0791	90°·0	·0623	·0791
15·200	22·687	31·034	·2851	·4272	·5845	·0565	·0788	·0785			
15·400	22·712	30·993	·2888	·4277	·5837	·0528	·0783	·0777			
15·600	22·762	30·952	·2926	·4286	·5830	·0490	·0774	·0770			
15·800	22·840	30·890	·2963	·4301	·5818	·0453	·0759	·0758			
16·000	22·925	30·826	·3001	·4317	·5806	·0415	·0743	·0746			
16·200	23·010	30·720	·3039	·4333	·5786	·0377	·0727	·0726	60°·0	·0370	·0720
16·400	23·128	30·590	·3076	·4356	·5762	·0340	·0704	·0702			
16·600	23·250	30·447	·3114	·4378	·5735	·0302	·0682	·0675			
16·800	23·418	30·274	·3151	·4410	·5702	·0265	·0650	·0642			
17·000	23·620	30·090	·3189	·4448	·5667	·0227	·0612	·0607	45°·0	·0238	·0618
17·200	23·860	29·865	·3226	·4493	·5625	·0190	·0567	·0565			
17·600	24·437	29·278	·3301	·4602	·5514	·0115	·0458	·0454	30°·0	·0120	·0462
18·000	25·390	28·316	·3376	·4782	·5333	·0040	·0278	·0273	15°·0	·0033	·0251
18·214	***	***	·3416	·5060	·5060	·0000	·0000	·0000			

Weight 4·57 grs. $\beta = 2{\cdot}334$ $a = 119{\cdot}6$ $\omega = 144^\circ{\cdot}48$ $b = 0{\cdot}09878$ in. Temp. 40° F.

Error in calculation of $V = +0{\cdot}000\,035$ cubic inch.

No. II

Original readings in turns of the screws			Readings converted into inches			The same when the origin of coordinates is at the vertex			Calculated		
z''	x_1''	x_2''	z'	x_1'	x_2'	z	x_1	x_2	ϕ	z	x
			inch	inch	inch	inch	inch	inch		inch	inch
13·023	24·710	34·330	0·2443	0·4654	0·6467	0·1054	0·0907	0·0906	148°·28	0·1054	0·09065
13·100	24·603	34·464	·2457	·4634	·6492	·1040	·0927	·0931	145°·0	·1044	·0921
13·200	24·489	34·596	·2476	·4612	·6517	·1021	·0949	·0956	140°·0	·1028	·0943
13·300	24·356	34·678	·2495	·4587	·6532	·1002	·0974	·0971	135°·0	·1009	·0963
13·400	24·260	34·762	·2513	·4569	·6548	·0984	·0992	·0987			
13·600	24·146	34·904	·2551	·4547	·6575	·0946	·1014	·1014	120°·0	·0938	·1018
13·800	24·040	35·010	·2588	·4527	·6595	·0909	·1034	·1034			
14·000	23·975	35·086	·2626	·4515	·6609	·0871	·1046	·1048			
14·400	23·867	35·178	·2701	·4495	·6626	·0796	·1066	·1065			
14·732	23·848	35·203	·2763	·4491	·6631	·0734	·1070	·1070	90°·0	·0734	·1070
15·000	23·866	35·192	·2813	·4495	·6629	·0684	·1066	·1068			
15·400	23·950	35·105	·2888	·4510	·6613	·0609	·1051	·1052			
15·800	24·072	34·989	·2963	·4533	·6591	·0534	·1028	·1030			
16·200	24·270	34·782	·3039	·4571	·6552	·0458	·0990	·0991	60°·0	·0463	·0993
16·600	24·534	34·515	·3114	·4620	·6501	·0383	·0941	·0940			
17·000	24·906	34·162	·3189	·4691	·6435	·0308	·0871	·0874	45°·0	·0315	·0878
17·400	25·361	33·685	·3264	·4776	·6345	·0233	·0785	·0784			
17·800	25·978	33·078	·3339	·4893	·6230	·0158	·0668	·0669	30°·0	·0170	·0687
18·000	26·368	32·688	·3376	·4966	·6157	·0121	·0595	·0596			
18·200	26·867	32·165	·3414	·5060	·6058	·0083	·0501	·0497			
18·400	27·540	31·478	·3451	·5187	·5929	·0046	·0374	·0368	15°·0	·0051	·0394
18·600	28·637	30·348	·3489	·5394	·5716	·0008	·0167	·0155			
18·642	***	***	·3497	·5561	·5561	·0000	·0000	·0000			

Weight 9·523 grs. $\beta = 6{\cdot}44$ $a = 126{\cdot}2$ $\omega = 148^\circ{\cdot}28$ $b = 0{\cdot}15976$ in. Temp. 37° F.
Error in calculation of $V \doteq +0{\cdot}000\,044$ cubic inch.

No. III

Original readings in turns of the screws			Readings converted into inches			The same when the origin of coordinates is at the vertex			Calculated		
z''	x_1''	x_2''	z'	x_1'	x_2'	z	x_1	x_2	ϕ	z	x
			inch	inch	inch	inch	inch	inch		inch	inch
12·570	22·563	34·683	0·2358	0·4249	0·6533	0·1127	0·1151	0·1133	141°·71	0·1127	0·1142
12·900	22.282	35·050	·2420	·4196	·6602	·1065	·1204	·1202	140°·00	·1121	·1150
13·000	22·232	35·112	·2438	·4187	·6614	·1047	·1213	·1214	135°·00	·1101	·1171
13·400	22·024	35·326	·2513	·4147	·6654	·0972	·1253	·1254	120°·00	·1026	·1228
14·300	21·863	35·480	·2682	·4117	·6683	·0803	·1283	·1283	90°·00	·0813	·1283
16·000	22.418	34·928	·3001	·4222	·6579	·0484	·1178	·1179	60°·00	·0527	·1202
17·000	23·358	33·985	·3189	·4399	·6401	·0296	·1001	·1001	45°·00	·0367	·1078
18·000	25·225	32·132	·3376	·4751	·6052	·0109	·0649	·0652	30°·00	·0206	·0865
18·580	***	***	·3485	·5400	·5400	·0000	·0000	·0000	15°·00	·0065	·0516

Weight 14·725 grs. $\beta = 11{\cdot}0$ $a = 118{\cdot}2$ $\omega = 141^\circ{\cdot}71$ $b = 0{\cdot}21572$ in. Temp. 39° F.

Error in calculation of $V = +0{\cdot}000\ 057$ cubic inch.

No. IV

Original readings in turns of the screws			Readings converted into inches			The same when the origin of coordinates is at the vertex			Calculated		
z''	x_1''	x_2''	z'	x_1'	x_2'	z	x_1	x_2	ϕ	z	x
			inch	inch	inch	inch	inch	inch		inch	inch
12·985	19·918	33·865	0·2435	0·3750	0·6379	0·1168	0·1311	0·1318	140°·0	0·1168	0·1315
13·000	19·900	33·918	·2438	·3747	·6389	·1165	·1314	·1328			
13·100	19·812	34·010	·2457	·3730	·6406	·1146	·1331	·1345	135°·00	·1148	·1337
13·200	19·725	34·093	·2476	·3714	·6422	·1127	·1347	·1361			
13·300	19·642	34·167	·2495	·3698	·6436	·1108	·1363	·1375			
13·400	19·553	34·230	·2513	·3681	·6448	·1090	·1380	·1387			
13·500	19·500	34·300	·2532	·3670	·6461	·1071	·1391	·1400	120°·00	·1072	·1394
13·600	19·440	34·362	·2551	·3660	·6472	·1052	·1401	·1411			
13·800	19·360	34·412	·2588	·3645	·6482	·1015	·1416	·1421			
14·000	19·294	34·486	·2626	·3633	·6496	·0977	·1428	·1435			
14·200	19·243	34·532	·2663	·3623	·6505	·0940	·1438	·1444			
14·400	19·195	34·548	·2701	·3614	·6508	·0902	·1447	·1447			
14·600	19·182	34·560	·2738	·3611	·6510	·0865	·1450	·1449	90°·00	·0856	·1450
14·800	19·200	34·560	·2776	·3615	·6510	·0827	·1446	·1449			
15·000	19·226	34·545	·2813	·3620	·6507	·0790	·1441	·1446			
15·200	19·277	34·520	·2851	·3629	·6502	·0752	·1432	·1441			
15·400	19·307	34·456	·2888	·3635	·6490	·0715	·1426	·1429			
15·600	19·353	34·388	·2926	·3644	·6477	·0677	·1417	·1416			
15·800	19·445	34·322	·2963	·3661	·6465	·0640	·1400	·1404			
16·000	19·539	34·228	·3001	·3679	·6447	·0602	·1382	·1386			
16·200	19·652	34·107	·3039	·3700	·6424	·0564	·1361	·1363	60°·00	·0567	·1367
16·400	19·760	33·975	·3076	·3721	·6399	·0527	·1340	·1338			
16·600	19·913	33·832	·3114	·3749	·6373	·0489	·1312	·1312			
16·800	20·050	33·687	·3151	·3775	·6345	·0452	·1286	·1284			
17·000	20·240	33·496	·3189	·3811	·6309	·0414	·1250	·1248	45°·00	·0402	·1239
17·200	20·442	33·282	·3226	·3849	·6269	·0377	·1212	·1208			
17·400	20·653	33·036	·3264	·3889	·6223	·0339	·1172	·1162			
17·600	20·902	32·792	·3301	·3936	·6177	·0302	·1125	·1116			
17·800	21·204	32·516	·3339	·3993	·6125	·0264	·1068	·1064			
18·000	21·548	32·178	·3376	·4058	·6061	·0227	·1003	·1000	30°·00	·0234	·1016
18·200	21·927	31·786	·3414	·4129	·5987	·0189	·0932	·0926			
18·400	22·342	31·320	·3451	·4208	·5899	·0152	·0853	·0838			
18·600	22·880	30·816	·3489	·4309	·5804	·0114	·0752	·0743			
18·700	23·170	30·494	·3507	·4363	·5744	·0096	·0698	·0683			
18·800	23·517	30·134	·3526	·4429	·5676	·0077	·0632	·0615	15°·00	·0078	·0631
18·900	23·950	29·762	·3545	·4510	·5606	·0058	·0551	·0545			
19·000	24·436	29·205	·3564	·4602	·5501	·0039	·0459	·0440			
19·100	25·165	28·522	·3582	·4739	·5372	·0021	·0322	·0311			
19·210	***	***	·3603	·5061	·5061	·0000	·0000	·0000	0·00	·0000	·0000

Weight 19·77 grs. $\beta = 17{\cdot}5$ $\alpha = 116{\cdot}9$ $\omega = 140^\circ{\cdot}00$ $b = 0{\cdot}27358$ in. Temp. 38° F.

Error in calculation of $V = +\,0{\cdot}000\ 012$ cubic inch.

No. V

Original readings in turns of the screws			Readings converted into inches			The same when the origin of coordinates is at the vertex			Calculated		
z''	x_1''	x_2''	z'	x_1'	x_2'	z	x_1	x_2	ϕ	z	x
			inch	inch	inch	inch	inch	inch		inch	inch
17·732	21·555	36·516	0·3326	0·4059	0·6878	0·1174	0·1411	0·1408	139°·41	0·1174	0·14095
17·800	21·492	36·603	·3339	·4047	·6895	·1161	·1423	·1425			
17·900	21·398	36·702	·3357	·4029	·6914	·1143	·1441	·1444	135°·0	·1156	.1429
18·000	21·333	36·782	·3376	·4017	·6929	·1124	·1453	·1459			
18·200	21·189	36·910	·3414	·3990	·6953	·1086	·1480	·1483			
18·400	21·068	37·005	·3451	·3967	·6971	·1049	·1503	·1501	120°·00	·1082	·1486
18·600	20·986	37·094	·3489	·3952	·6987	·1011	·1518	·1517			
19·000	20·910	37·200	·3564	·3937	·7007	·0936	·1533	·1537			
19·336	20·868	37·214	·3627	·3929	·7010	·0873	·1541	·1540	90°·00	·0868	·1540
19·800	20·905	37·194	·3714	·3936	·7006	·0786	·1534	·1536			
20·209	21·006	37·095	·3789	·3955	·6988	·0711	·1515	·1518			
20·600	21·141	36·938	·3864	·3981	·6958	·0636	·1489	·1488			
21·000	21·368	36·736	·3939	·4024	·6920	·0561	·1446	·1450	60°·00	·0581	·1459
21·400	21·638	36·443	·4014	·4075	·6865	·0486	·1395	·1395			
21·800	22·000	36·084	·4089	·4143	·6797	·0411	·1327	·1327	45°·00	·0417	·1331
22·200	22·460	35·640	·4164	·4230	·6713	·0336	·1240	·1243			
22·600	23·031	35·052	·4239	·4337	·6603	·0261	·1133	·1133	30°·00	·0247	·1106
23·000	23·788	34·312	·4314	·4480	·6463	·0186	·0990	·0993			
23·400	24·786	33·256	·4389	·4668	·6264	·0111	·0802	·0794	15°·00	·0086	·0707
23·800	26·546	31·570	·4463	·5000	·5946	·0037	·0470	·0476			
23·995	***	***	·4500	·5470	·5470	·0000	·0000	·0000	0	0	0

Weight ? $\beta = 24{\cdot}023$ $a = 119{\cdot}9$ $\omega = 139^{\circ}{\cdot}41$ $b = 0{\cdot}31646$ in. Temp. 49° F.

The theoretical forms of these five drops of mercury have been drawn to a large scale in Fig. 2, where the measured points are indicated by small crosses.

The agreement between theory and experiment appears to be so far satisfactory. And if on more exact comparison any slight discrepancy between theory and experiment should become apparent, it will be known that this is not due to any error in the calculated forms.

In adapting a theoretical form to the measured form of a drop of mercury, it would be sufficient to secure its passing through the vertex A (Fig. 4) and the two points B, C, for which $\phi = 90^\circ$, if it was possible to measure AO *correctly*. But this can be accomplished practically only with sufficient accuracy to give a rough first approximation to the value of β, by finding $OC \div AO$ and referring to Table I. This value of β, if erroneous, must be corrected by trial till a curve is found from Table II., which passes through D and E, the extremities of the base, or till two curves are found for consecutive values of β, one of which falls outside, and the other within DE. Then by proportional parts the exact value of β required can be found.

Let $BC = 2R$, $DE = 2r$, and $AN = H$. The following example will explain how the values of the capillary constants are obtained by means of these quantities.

For the drop No. V. $2R = 0{\cdot}3081$ inch, $H = 0{\cdot}1174$ inch, and $2r = 0{\cdot}2819$ inch. Having found by the help of Table I. and by trial that the proper value of β lies between 24·0 and 24·1, we proceed to find b' the radius of curvature at the vertex corresponding to $\beta' = 24{\cdot}0$. From Table II., when $\phi = 90^\circ$, we find that $\frac{x}{b'} = \frac{R}{b'} = \frac{0{\cdot}15405}{b'} = 0{\cdot}48692$ and therefore $b' = \frac{0{\cdot}15405}{0{\cdot}48692}$, which gives $\log b' = 9{\cdot}50020$. That will suffice to secure a curve which passes through the vertex A and has the correct width BC. We wish in addition to secure a curve which passes through the two points D, E at the base of the drop, or through two points d, e near the base.

$$\begin{array}{ll} \log r = 9{\cdot}14907 & \log H = 9{\cdot}06967 \\ \log b' = 9{\cdot}50020 & \log b' = 9{\cdot}50020 \\ \hline \log \frac{r}{b'} = \overline{9}{\cdot}64887 & \log \frac{H}{b'} = \overline{9}{\cdot}56947 \end{array}$$

and therefore $\frac{r}{b'} = 0{\cdot}44552$ and $\frac{H}{b'} = 0{\cdot}37108$.

And to find the theoretical form of this drop we use the manuscript Tables above referred to[a]. For $\beta' = 24{\cdot}0$ the Table gives $\frac{z}{b'} = 0{\cdot}37108 = \frac{H}{b'}$ corresponding to $\phi = 139^\circ{\cdot}36$. And for the same value of ϕ, $\frac{x}{b'} = 0{\cdot}44608 - 0{\cdot}00050 = 0{\cdot}44558$. Hence $\frac{H}{b'} - \frac{z}{b'} = 0$ and $\frac{r}{b'} - \frac{x}{b'} = -0{\cdot}00006$, $\phi = 139^\circ{\cdot}36$.

[a] See note [a] on next page.

Again, corresponding to $\beta''=24\cdot1$ we find in the same manner as before $\log b''=9\cdot50070$, which gives $\frac{r}{b''}=0\cdot44501$ and $\frac{H}{b''}=0\cdot37066$. And for $\beta''=24\cdot1$[b] the Table gives $\frac{z}{b''}=0\cdot37066$ corresponding to $\phi=139^\circ\cdot58$, and $\frac{x}{b''}=0\cdot44481$. Here we have

$$\frac{H}{b''}-\frac{z}{b''}=0\,;\ \frac{r}{b''}-\frac{x}{b''}=+0\cdot00020\,;\ \phi=139^\circ\cdot58.$$

The required value of β therefore falls between 24·0 and 24·1, and its exact value may be found as follows:

$\beta'=24\cdot0$ gives	$\log b'=9\cdot50020$;	error in $\frac{x}{b'}=-0\cdot00006$;	and $\phi'=139^\circ\cdot36$
$\beta''=24\cdot1$ gives	$\log b''=9\cdot50070$;	„ $\frac{x}{b''}=+0\cdot00020$;	and $\phi''=139\cdot58$
Diff. $+\ 0\cdot1$	$+0\cdot00050$	$+0\cdot00026$	$+0\cdot22$

Hence by taking proportional parts we must find such values of $\delta\beta'$, $\delta\log b'$ and $\delta\phi'$ as will make the error in $\frac{x}{b'}$ vanish.

$\beta'=24\cdot0$	gives $\log b'=9\cdot50020$;	error in $\frac{x}{b'}=-0\cdot00006$;	and $\phi'=139^\circ\cdot36$
$\delta\beta'=+\cdot023$	„ $\delta\log b'=+0\cdot00012$;	„ „ $+0\cdot00006$;	„ $\delta\phi'=+0\cdot05$
Hence $\beta=24\cdot023$	„ $\log b=9\cdot50032$;	„ „ 0	„ $\phi=139\cdot41$

Hence $b=0\cdot31646$ inch, $\alpha=\frac{\beta}{2b^2}=\frac{12\cdot0115}{b^2}=119\cdot94$, and $\sqrt{\frac{2}{\alpha}}=0\cdot1291$ inch. Also the volume $=b^3\times0\cdot2072=0\cdot0065667$ cubic inch, and the corresponding weight is 22·535 grains.

Nearly the same results might be arrived at by using the values of $\frac{x}{b}$ and $\frac{z}{b}$ in Table II. for $\beta=24$ and $\beta=25$, only the differences would correspond to a difference of 1 instead of 0·1 in the value of β, and to a difference of 5° instead of 1° in the value of ϕ.

NOTE [a].

$\beta=24\cdot0$				
ϕ	$\frac{x}{b}$	Δ	$\frac{z}{b}$	Δ
...	......		......	
138°	·44747	−139	·36942	+123
139	·44608	−140	·37065	+120
140	·44468		·37185	
&c.	&c.		&c.	

NOTE [b].

$\beta=24\cdot1$				
ϕ	$\frac{x}{b}$	Δ	$\frac{z}{b}$	Δ
...	......		......	
138°	·44700	−139	·36874	+123
139	·44561	−140	·36997	+120
140	·44421		·37117	
&c.	&c.		&c.	

It is evident that the above calculations would have been facilitated if the Tables referred to in the note had been calculated for $\log \frac{x}{b}$, $\log \frac{z}{b}$ and $\log \frac{V}{b^3}$ rather than for $\frac{x}{b}$, $\frac{z}{b}$ and $\frac{V}{b^3}$, as has been already remarked.

The coordinates at the points of the theoretical curve at which the tangent is inclined to the horizon at angles of 15°, 30°, 45°, 60°, 90°, 120°, 135° &c., are found by the help of Table V. for values of β, 0·0, 0·1, 0·2, 0·3 ...... 46·5, 46·6, 46·7.

For instance for $\phi = 135°$,

$$\begin{array}{llll} \beta' = 24{\cdot}0 \;; & & \frac{x'}{b} = 0{\cdot}45156\;; & \frac{z'}{b} = 0{\cdot}36554 \\ +\delta\beta' = 0{\cdot}023 & \text{gives} & -11 & -15 \\ \beta = \overline{24{\cdot}023}\;; & & \frac{x}{b} = \overline{0{\cdot}45145}\;; & \frac{z}{b} = \overline{0{\cdot}36539} \end{array}$$

and $b = 0{\cdot}31646$ inch.

Therefore $x = b \times 0{\cdot}45145 = 0{\cdot}1429$ inch,

and $z = b \times 0{\cdot}36539 = 0{\cdot}1156$ inch.

DETERMINATION OF CAPILLARY CONSTANTS OF MERCURY IN CONTACT WITH GLASS.

The great impediment to the exact determination of capillary constants arises from the changes that usually take place at capillary surfaces when left undisturbed for some time. All careful experimenters have recognised this difficulty. It seemed therefore best to place a drop of mercury in position and to take measures of $2R$, $2r$ and H as opportunity offered. Drops weighing 4, 8, 12, 16, 20 and 24 grains were used, because it was expected that, if α and ω were not really constant for mercury resting on glass, some indication of the manner in which they varied would thus be made manifest. The mercury was obtained as being pure from a leading philosophical instrument maker about 1862. When any experiment was to be made, a sufficient quantity was taken from this store, and after having been used, it was treated as waste. Also the same glass plate table was used in all the experiments. The glass plate was cleaned with blotting paper or with the pith of the stalk of the artichoke. And after this, either the same or a fresh drop of mercury was placed in position and vibrated. In the following tables of experiments the operation of cleaning the glass and replacing the *same* drop of mercury is indicated by a dotted line ...... But a change in the mercury used is denoted by a line — across the table. The reading of the thermometer is given and also the time during which the drop had been in position when the measurement was made. The experiments were carried on in a small workshop built in a garden apart from other buildings. The

observing table rested on supports driven into the ground which were independent of the brick floor. There were public roads, used chiefly for light traffic, on two sides at the distances of 50 and 60 yards. The slow changes in the forms of drops of fluid appear to arise, (1) from some small change that takes place in the tension of the enveloping surface, (2) from changes of temperature between night and day, and (3) from slight tremors arising from passing vehicles, &c. The calculation of the capillary constants was carried on as the experiments were made. After all had been completed the reductions of the instrumental observations into inches were carefully examined, and the calculations of all the 145 experiments were repeated, so that the results given in the following tables may be considered to be quite correct.

The variation in the value of the capillary constants deduced from drops of mercury of the same size was much greater than was expected. But, when the mean values of ω and α derived from each form of drop were compared, the agreement was surprisingly close. Hence so far as these experiments go the form of sessile drops appears to be that indicated by the Theories of Young, Laplace, Gauss and Poisson.

Finally the values of α, ω, and V were calculated from the mean values of $2R$, $2r$, and H for each size of drop of mercury. The results are given on the last page for comparison with the means of the values of α and ω derived from each experiment for each size of drop of mercury.

In order to carry out the original scheme, as sketched in the Introduction, many more experiments should be made, particularly for the purpose of finding the effects of variation of temperature on the values of the capillary constants.

The calculations for negative values of β should also be greatly extended, so that the intervals between them might be readily filled up by interpolation, as we have done in the case of positive values of β.

The measuring instrument in its present form appears to be satisfactory. The microscope descends in a vertical direction by its own weight and is raised by the screw. A screw of about 50 turns to the inch is very suitable for experimenting with mercury. But a quicker motion will become desirable when experiments are made with a drop of one fluid immersed in another fluid, as the drops may be then much larger.

All documents connected with these calculations now in my possession will be carefully preserved, and every assistance will be afforded to any person who may undertake the completion of the work.

MINTING VICARAGE, *Oct.* 1883.

Drop of 4 Grains of Mercury.

No. of Observation	R	H	r	β	a	ω	$\sqrt{\frac{2}{a}}$		Temp. F	Error in V	Hours in position
	inch	inch	inch			°	inch	m. metres		cubic inch	
1	0·07460	0·09050	0·05990	1·925	117·00	145·43	0·1308	3·321	60°	+ ·000002	?
24	·07535	·09150	·06040	1·922	114·54	145·65	·1321	3·356	61	+ ·000039	0
25	·07635	·08950	·06170	2·394	130·70	146·67	·1237	3·142	58	+ ·000045	12
26	·07610	·08940	·06185	2·316	128·52	145·48	·1248	3·169	59	+ ·000036	18
27	·07600	·08950	·06165	2·292	127·90	145·68	·1251	3·176	61	+ ·000034	?
28	·07475	·09040	·05965	2·005	120·03	146·70	·1291	3·279	60	+ ·000005	½
29	·07570	·08870	·06070	2·457	135·42	148·11	·1215	3·087	58	+ ·000013	13
30	·07580	·08930	·06050	2·400	132·84	148·52	·1227	3·117	60	+ ·000022	8
31	·07565	·08895	·06045	2·424	134·18	148·47	·1221	3·101	62	+ ·000014	22
117	·07525	·08950	·06115	2·126	123·59	144·68	·1272	3·231	63	+ ·000010	1
118	·07595	·08760	·06140	2·670	142·61	147·73	·1184	3·008	62	+ ·000005	13
119	·07585	·08790	·06160	2·549	138·41	146·56	·1202	3·053	64	+ ·000007	15
120	·07585	·08875	·06090	2·472	135·47	147·97	·1215	3·086	66	+ ·000017	3
121	·07535	·08795	·05940	2·638	143·56	151·40	·1180	2·997	64	− ·000009	34
122	·07545	·08800	·05965	2·633	143·09	150·96	·1182	3·003	67	− ·000005	36
123	·07620	·08760	·06140	2·755	144·77	148·57	·1175	2·985	66	+ ·000014	21
124	·07380	·08850	·05860	2·157	129·85	148·16	·1241	3·152	63	− ·000048	0
125	·07430	·08810	·05960	2·263	132·60	147·20	·1228	3·119	62	− ·000037	24
126	·07475	·08770	·05955	2·481	139·85	149·17	·1196	3·038	61	− ·000030	48
127	·07490	·08760	·05975	2·525	141·02	149·16	·1191	3·025	63	− ·000027	71
128	·07470	·08925	·05970	2·172	127·39	147·32	·1253	3·183	61	− ·000011	88
129	·07460	·08970	·05965	2·075	123·57	146·77	·1272	3·231	61	− ·000008	0
130	·07480	·08940	·06035	2·104	124·15	145·62	·1269	3·224	61	− ·000005	38
131	·07515	·08880	·06040	2·305	131·33	147·09	·1234	3·134	59	− ·000002	61
132	·07545	·08880	·06045	2·391	133·73	147·94	·1223	3·106	57	+ ·000007	96
133	·07550	·08850	·06040	2·467	136·53	148·49	·1210	3·074	58	+ ·000005	111
Means	0·07531	0·08890	0·06041		132·03	147·52	0·1233	3·131			

Drop of 8 Grains of Mercury.

No. of Observation	R	H	r	β	a	ω	$\sqrt{\frac{2}{a}}$		Temp. F	Error in V	Hours in position
	inch	inch	inch			°	inch	m. metres		cubic inch	
2	0·09980	0·10100	0·08535	5·226	127·96	144·45	0·1250	3·175	60°	+ ·000001	?
3	·10000	·10090	·08535	5·370	129·57	145·10	·1242	3·156	60	+ ·000017	?
4	·09905	·10360	·08345	4·456	117·69	145·89	·1304	3·311	58	+ ·000030	?
32	·09915	·10210	·08380	4·878	124·28	146·11	·1268	3·222	62	+ ·000004	1
33	·09990	·10070	·08400	5·700	134·61	148·62	·1219	3·096	61	+ ·000008	18
34	·09990	·10110	·08410	5·534	132·23	148·09	·1230	3·124	62	+ ·000017	22
134	·09945	·10295	·08315	4·892	123·74	148·18	·1271	3·229	58	+ ·000037	5
135	·10010	·10180	·08350	5·521	131·53	149·83	·1233	3·132	57	+ ·000043	21
136	·10015	·10180	·08340	5·567	132·05	150·25	·1231	3·126	57	+ ·000046	29
137	·09940	·10290	·08320	4·874	123·59	147·94	·1272	3·231	58	+ ·000034	12
138	·09975	·10265	·08330	5·101	126·22	148·82	·1259	3·197	58	+ ·000045	22
139	·09950	·10230	·08335	5·079	126·52	148·19	·1257	3·194	56	+ ·000026	0
140	·09960	·10200	·08335	5·228	128·51	148·67	·1248	3·169	58	+ ·000023	10
141	·09960	·10150	·08390	5·280	129·29	147·51	·1244	3·159	55	+ ·000013	13
142	·09980	·10120	·08420	5·423	130·89	147·44	·1236	3·140	56	+ ·000016	23
143	·09985	·10100	·08445	5·464	131·35	147·06	·1234	3·134	57	+ ·000014	37
144	·09985	·10090	·08435	5·522	132·20	147·39	·1230	3·124	58	+ ·000011	46
145	·09965	·10120	·08430	5·321	129·77	146·75	·1241	3·153	58	+ ·000008	61
Means	0·09969	0·10176	0·08392		128·44	147·57	0·1248	3·171			

Drop of 12 Grains of Mercury.

No. of Observation	R	H	r	β	a	ω	$\sqrt{\frac{2}{a}}$		Temp. F	Error in V	Hours in position
	inch	inch	inch			°	inch	m.metres		cubic inch	
5	0·11720	0·10900	0·10110	8·728	124·99	147·27	0·1265	3·213	59°	− ·000001	?
6	·11735	·10920	·10230	8·331	121·50	144·48	·1283	3·259	59	+ ·000011	2
7	·11763	·10785	·10288	9·044	126·53	144·43	·1257	3·193	59	− ·000013	20
35	·11700	·11020	·10020	8·267	121·70	148·32	·1282	3·256	62	+ ·000022	½
36	·11810	·10700	·10115	10·592	136·68	150·66	·1210	3·073	64	− ·000009	38
37	·11830	·10800	·10105	10·183	133·39	150·91	·1225	3·110	63	+ ·000035	2
38	·11810	·10785	·10115	10·073	133·07	150·20	·1226	3·114	64	+ ·000016	9
39	·11710	·11030	·10050	8·200	120·94	147·83	·1286	3·266	62	+ ·000032	¼
40	·11810	·10750	·10135	10·215	134·08	149·89	·1221	3·102	63	+ ·000006	12
41	·11770	·10790	·10135	9·642	130·85	148·58	·1236	3·140	62	− ·000005	13
42	·11770	·10760	·10160	9·726	131·47	148·10	·1233	3·133	64	− ·000015	16
43	·11740	·10955	·10005	8·925	126·11	150·17	·1259	3·199	63	+ ·000027	2
44	·11675	·10915	·10025	8·588	124·84	148·15	·1266	3·215	63	− ·000025	7
45	·11710	·10787	·10035	9·509	131·22	149·54	·1235	3·136	63	− ·000043	21
46	·11740	·10820	·10040	9·546	130·81	150·04	·1237	3·141	64	− ·000014	45
47	·11765	·10790	·10025	9·969	133·35	151·26	·1225	3·111	65	− ·000009	145
Means	0·11754	0·10844	0·10100		128·85	148·74	0·1247	3·166			

Drop of 16 Grains of Mercury.

No. of Observation	R	H	r	β	a	ω	$\sqrt{\frac{2}{a}}$		Temp. F	Error in V	Hours in position
	inch	inch	inch			°	inch	m.metres		cubic inch	
8	0·13298	0·11305	0·11645	14·703	127·57	148·23	0·1252	3·180	59°	+ ·000010	6
9	·13290	·11283	·11642	14·809	128·19	148·20	·1249	3·173	59	+ ·000012	7
10	·13185	·11370	·11535	13·448	124·10	147·70	·1270	3·225	55	− ·000031	0
11	·13215	·11290	·11575	14·228	127·08	147·90	·1255	3·187	51	− ·000033	2
48*	·13275	·11420	·11675	13·355	123·77	146·32	·1271	3·229	67	+ ·000014	½
49*	·13378	·11250	·11750	15·584	129·73	147·94	·1242	3·154	66	+ ·000028	3
50*	·13375	·11245	·11760	15·530	129·57	147·63	·1242	3·156	66	+ ·000021	6
51*	·13315	·11285	·11745	14·529	126·50	146·25	·1257	3·194	65	− ·000008	11
52*	·13338	·11240	·11725	15·311	129·38	147·57	·1243	3·158	63	− ·000007	21
53	·13225	·11525	·11718	11·853	115·62	143·46	·1315	3·341	62	+ ·000053	½
54	·13415	·11115	·11823	16·922	134·30	147·72	·1220	3·100	62	+ ·000037	47
55	·13338	·11240	·11695	15·472	130·06	148·31	·1240	3·150	63	+ ·000028	16
56	·13325	·11210	·11665	15·758	131·48	148·89	·1233	3·133	63	+ ·000009	19
Means	0·13306	0·11291	0·11689		127·49	147·39	·1253	3·183			

* Weight of this drop was 16·12 grains.

Drop of 20 Grains of Mercury.

No. of Observation	R	H	r	β	a	ω	$\sqrt{\frac{2}{a}}$		Temp. F	Error in V	Hours in position
	inch	inch	inch			°	inch	m. metres		cubic inch	
12	0·14655	0·11425	0·13040	24·243	133·07	147·93	0·1226	3·114	51°	− ·000033	12
13	·14405	·11910	·12710	17·063	116·94	147·47	·1308	3·322	51	− ·000013	2
14	·14420	·11880	·12740	17·373	117·71	147·28	·1304	3·311	51	− ·000016	3
15	·14325	·12100	·12605	15·018	111·10	147·16	·1342	3·408	51	+ ·000002	?
16	·14420	·11810	·12680	18·438	121·11	149·04	·1285	3·264	51	− ·000046	?
17	·14435	·11820	·12705	18·387	120·69	148·76	·1287	3·270	52	− ·000031	?
57	·14563	·11725	·12980	19·386	121·60	145·71	·1283	3·258	61	+ ·000020	½
58	·14590	·11695	·13023	19·836	122·44	145·50	·1278	3·246	61	+ ·000027	1
59	·14803	·11370	·13200	26·497	135·72	148·00	·1214	3·083	62	+ ·000054	47
60	·14458	·11785	·12720	18·967	122·11	149·11	·1280	3·251	62	− ·000036	½
61	·14508	·11665	·12788	20·595	126·03	149·26	·1260	3·200	63	− ·000048	13
62	·14528	·11730	·12785	20·184	124·50	149·58	·1267	3·219	61	+ ·000001	4
63	·14543	·11685	·12785	20·937	126·38	150·17	·1258	3·195	63	− ·000008	19
64	·14520	·11695	·12755	20·671	126·03	150·27	·1260	3·200	64	− ·000023	24
65	·14513	·11695	·12790	20·330	125·18	149·25	·1264	3·211	63	− ·000029	29
66	·14650	·11580	·13045	22·031	127·50	146·98	·1252	3·181	63	+ ·000027	1
67	·14775	·11380	·13193	25·840	134·71	147·38	·1218	3·095	61	+ ·000037	12
68	·14790	·11360	·13220	26·186	135·24	147·17	·1216	3·089	60	+ ·000040	23
69	·14805	·11380	·13160	26·769	136·29	148·99	·1211	3·077	61	+ ·000061	13
70	·14758	·11480	·13233	23·662	129·80	145·48	·1241	3·153	64	+ ·000063	25
71	·14768	·11472	·13223	24·065	130·62	146·04	·1237	3·143	63	+ ·000068	36
72	·14618	·11535	·12905	23·175	131·05	149·82	·1235	3·138	65	− ·000020	3
73	·14615	·11530	·12935	22·958	130·54	149·04	·1238	3·144	65	− ·000023	7
74	·14675	·11382	·13028	25·358	135·42	148·94	·1215	3·087	66	− ·000046	47
75	·14648	·11505	·13043	22·982	130·02	147·31	·1240	3·150	66	− ·000011	49
76	·14640	·11550	·13015	22·481	128·86	147·59	·1246	3·164	67	·000000	53
Means	0·14593	0·11621	·12935		126·95	148·05	0·1256	3·191			

Drop of 24 Grains of Mercury.

No. of Observation	R	H	r	β	a	ω	$\sqrt{\frac{2}{a}}$		Temp. F	Error in V	Hours in position
	inch	inch	inch			°	inch	m.metres		cubic inch	
18	0·16085	0·11800	0·14550	33·541	127·29	145·29	0·1254	3·184	54°	+ ·000241	22
19	·16055	·11840	·14550	32·000	125·23	144·36	·1264	3·210	55	+ ·000231	23
20	·16060	·11820	·14570	32·233	125·47	144·07	·1263	3·207	58	+ ·000225	35
21	·15870	·11825	·14085	32·264	130·29	151·15	·1239	3·147	56	+ ·000072	17
22	·15638	·12010	·13745	28·249	125·08	152·70	·1265	3·212	60	− ·000050	8
23	·15645	·11925	·13730	29·889	128·08	153·63	·1250	3·174	60	− ·000078	32
77	·15835	·11830	·14243	30·584	126·26	146·41	·1259	3·197	67	+ ·000024	½
78	·15970	·11583	·14380	37·232	134·90	147·49	·1218	3·093	65	+ ·000014	14
79	·15923	·11582	·14453	34·856	132·01	144·44	·1231	3·126	65	− ·000039	24
80	·15923	·11685	·14455	32·750	128·58	143·97	·1247	3·168	66	+ ·000016	38
81	·15845	·11770	·14215	32·230	128·97	147·64	·1245	3·163	66	− ·000001	3
82	·15893	·11710	·14248	34·217	131·48	148·28	·1233	3·133	65	+ ·000008	8
83	·15908	·11615	·14210	37·078	135·72	150·00	·1214	3·083	63	− ·000020	19
84	·15898	·11610	·14208	36·928	135·67	149·82	·1214	3·084	63	− ·000033	31
85	·15675	·11950	·14013	27·527	123·08	147·51	·1275	3·238	64	− ·000061	½
86	·15840	·11710	·14280	32·457	129·43	146·13	·1243	3·157	64	− ·000038	16
87	·15908	·11675	·14323	34·352	131·45	146·95	·1234	3·133	65	+ ·000005	24
88	·15945	·11570	·14325	37·580	135·84	148·28	·1213	3·082	63	− ·000010	60
89	·15938	·11670	·14343	34·991	131·99	147·22	·1231	3·127	62	+ ·000032	72
90	·15790	·11900	·14320	27·462	121·18	143·09	·1285	3·263	61	+ ·000009	86
91	·15855	·11825	·14330	30·109	125·10	144·80	·1264	3·212	61	+ ·000037	87
92	·15683	·11947	·13950	28·350	124·51	149·24	·1267	3·219	61	− ·000049	1
93	·15660	·11895	·13985	28·357	124·94	148·05	·1265	3·214	61	− ·000098	15
94	·15640	·11930	·13990	27·340	123·28	147·28	·1274	3·235	61	− ·000100	17
95	·15795	·11805	·14040	32·352	130·00	150·46	·1240	3·151	61	− ·000018	6
96	·15805	·11750	·14065	33·373	131·55	150·37	·1233	3·132	64	− ·000037	26
97	·15780	·11815	·14053	31·695	129·10	149·76	·1245	3·161	64	− ·000033	31
98	·15750	·11775	·14065	31·592	129·42	148·88	·1243	3·158	62	− ·000081	45
99	·15820	·11830	·14220	30·500	126·35	146·60	·1258	3·196	63	+ ·000012	4
100	·15760	·11845	·14120	29·962	126·35	147·47	·1258	3·196	65	− ·000040	1
101	·15785	·11810	·14163	30·681	127·24	147·21	·1254	3·184	67	− ·000033	7
102	·15820	·11755	·14195	32·145	129·23	147·53	·1244	3·160	61	− ·000027	22
103	·15830	·11755	·14200	32·333	129·39	147·66	·1243	3·158	62	− ·000020	31
104	·15745	·11790	·14095	30·860	128·21	147·94	·1249	3·172	62	− ·000076	½
105	·15785	·11785	·14145	31·339	128·40	147·76	·1248	3·170	63	− ·000040	12
106	·15730	·11850	·14120	29·209	125·44	146·69	·1263	3·207	63	− ·000063	26
107	·15810	·11795	·14250	30·517	126·54	145·75	·1257	3·193	64	− ·000020	36
108	·15820	·11740	·14200	32·372	129·62	147·48	·1242	3·155	61	− ·000033	50
109	·15785	·11740	·14210	31·391	128·49	146·34	·1248	3·169	61	− ·000072	74
110	·15770	·11775	·14220	30·263	126·73	145·54	·1256	3·191	61	− ·000066	98
111	·15765	·11760	·14225	30·348	126·96	145·37	·1255	3·188	63	− ·000081	146
112	·15770	·11870	·14120	29·761	125·82	147·61	·1261	3·202	64	− ·000013	1
113	·15850	·11710	·14250	33·077	130·32	147·13	·1239	3·147	61	− ·000025	10
114	·15820	·11710	·14220	32·692	130·16	147·11	·1240	3·149	58	− ·000051	34
115	·15830	·11700	·14210	33·261	130·95	147·66	·1236	3·139	61	− ·000045	59
116	·15850	·11725	·14260	32·668	129·63	146·82	·1242	3·155	61	− ·000019	81
Means	0·15825	0·11778	0·14199		128·52	147·46	0·1248	3·169			

Summary of Mean Results for each weight of Drop of Mercury.

Weight of Drop	Laplace's a	Error in a	ω	Error in ω	$\sqrt{\frac{2}{a}}$	Error	$\sqrt{\frac{2}{a}}$	Error
Grains			°	°	inch	inch	m. metres	m. metre
4	132·02	+3·31	147·52	−0·27	0·1233	−·0015	3·131	−0·038
8	128·44	−0·27	147·57	−0·22	·1248		3·171	+0·002
12	128·85	+0·14	148·74	+0·95	·1247	−·0001	3·166	−0·003
16	127·49	−1·22	147·39	−0·40	·1253	+·0005	3·183	+0·014
20	126·95	−1·76	148·05	+0·26	·1256	+·0008	3·191	+0·022
24	128·52	−0·19	147·46	−0·33	·1248		3·169	
Means	128·71		147·79		0·1248		3·169	

Values of a, ω, &c., deduced from the mean values of R, H, and r for each size of Drop of Mercury.

Weight of Drop	R	H	r	β	a	ω	$\sqrt{\frac{2}{a}}$		Error in V
Grains	inch	inch	inch			°	inch	m. metres	cubic inch
4	0·07531	0·08890	0·06041	2·334	131·96	147·51	0·1231	3·127	+0·000003
8	·09969	·10176	·08392	5·236	128·41	147·56	·1248	3·170	+ ·000021
12	·11754	·10844	·10100	9·328	128·88	148·76	·1246	3·164	+ ·000002
16	·13306	·11291	·11689	14·681	127·32	147·40	·1253	3·183	+ ·000025
20	·14593	·11621	·12935	21·433	126·87	148·07	·1256	3·189	+ ·000001
24	·15825	·11778	·14199	31·796	128·55	147·46	·1247	3·168	− ·000013
Means					128·67	147·79	0·1247	3·167	

The forms of these six drops are given in Fig. 3 on a large scale.

$$\left(\frac{x}{z}\right)_{\phi=90^\circ}$$

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
0	1·00000	·02180	·04149	·05942	·07589	·09115	·10542	·11880	·13140	·14333
1	·15466	·16546	·17576	·18562	·19508	·20418	·21294	·22138	·22953	·23742
2	·24507	·25248	·25967	·26666	·27345	·28006	·28650	·29278	·29890	·30488
3	1·31072	·31643	·32201	·32748	·33283	·33807	·34320	·34824	·35318	·35803
4	·36278	·36745	·37204	·37656	·38100	·38535	·38963	·39386	·39802	·40211
5	·40615	·41012	·41403	·41789	·42169	·42544	·42914	·43278	·43638	·43993
6	1·44344	·44690	·45032	·45369	·45702	·46032	·46358	·46679	·46996	·47310
7	·47621	·47928	·48232	·48533	·48830	·49124	·49415	·49703	·49988	·50270
8	·50550	·50827	·51101	·51371	·51640	·51906	·52169	·52430	·52689	·52946
9	1·53200	·53452	·53702	·53949	·54194	·54437	·54678	·54917	·55154	·55389
10	·55621	·55851	·56080	·56307	·56533	·56758	·56981	·57202	·57421	·57638
11	·57852	·58065	·58277	·58488	·58698	·58906	·59112	·59317	·59520	·59722
12	1·59923	·60122	·60320	·60517	·60712	·60906	·61099	·61290	·61480	·61669
13	·61856	·62042	·62227	·62411	·62594	·62776	·62957	·63136	·63314	·63491
14	·63667	·63842	·64016	·64189	·64361	·64532	·64702	·64871	·65039	·65206
15	1·65372	·65537	·65701	·65864	·66027	·66189	·66350	·66510	·66669	·66827
16	·66984	·67140	·67296	·67451	·67605	·67758	·67910	·68062	·68213	·68363
17	·68512	·68661	·68809	·68956	·69102	·69248	·69393	·69537	·69681	·69824
18	1·69966	·70108	·70249	·70389	·70528	·70667	·70805	·70943	·71080	·71217
19	·71353	·71488	·71623	·71757	·71890	·72023	·72155	·72287	·72418	·72548
20	·72678	·72807	·72936	·73064	·73192	·73319	·73446	·73572	·73698	·73823
21	1·73947	·74071	·74194	·74317	·74440	·74562	·74684	·74805	·74926	·75046
22	·75165	·75284	·75403	·75521	·75639	·75756	·75873	·75989	·76105	·76221
23	·76336	·76451	·76565	·76679	·76792	·76905	·77017	·77129	·77241	·77352
24	1·77463	·77574	·77684	·77794	·77903	·78011	·78119	·78227	·78335	·78443
25	·78550	·78657	·78764	·78870	·78975	·79080	·79185	·79289	·79393	·79497
26	·79600	·79703	·79806	·79908	·80010	·80112	·80213	·80314	·80415	·80515
27	1·80615	·80715	·80814	·80913	·81012	·81110	·81208	·81306	·81404	·81501
28	·81598	·81695	·81791	·81887	·81983	·82078	·82173	·82268	·82362	·82456
29	·82550	·82643	·82736	·82829	·82922	·83015	·83107	·83199	·83291	·83383
30	1·83474	·83565	·83656	·83746	·83836	·83926	·84015	·84104	·84193	·84282
31	·84371	·84459	·84547	·84635	·84722	·84809	·84896	·84983	·85070	·85156
32	·85242	·85328	·85414	·85499	·85584	·85669	·85754	·85838	·85922	·86006
33	1·86090	·86173	·86256	·86339	·86422	·86505	·86587	·86669	·86751	·86833
34	·86915	·86996	·87077	·87158	·87239	·87320	·87400	·87480	·87560	·87640
35	·87719	·87798	·87877	·87956	·88035	·88113	·88191	·88269	·88347	·88425

I

$\left(\frac{x}{z}\right)\ \phi=90^\circ$

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
36	1·88503	·88580	·88657	·88734	·88811	·88888	·88964	·89040	·89116	·89192
37	·89268	·89344	·89419	·89494	·89569	·89644	·89719	·89793	·89867	·89941
38	·90015	·90089	·90163	·90236	·90309	·90382	·90455	·90528	·90600	·90672
39	1·90744	·90816	·90888	·90960	·91031	·91102	·91173	·91244	·91315	·91386
40	·91457	·91527	·91597	·91667	·91737	·91807	·91877	·91947	·92016	·92085
41	·92154	·92223	·92292	·92361	·92429	·92497	·92565	·92633	·92701	·92769
42	1·92836	·92904	·92971	·93038	·93105	·93172	·93239	·93306	·93372	·93438
43	·93504	·93570	·93636	·93702	·93768	·93833	·93898	·93963	·94028	·94093
44	·94158	·94223	·94288	·94352	·94416	·94480	·94544	·94608	·94672	·94735
45	1·94798	·94861	·94924	·94987	·95050	·95113	·95176	·95239	·95302	·95364
46	·95426	·95488	·95550	·95612	·95674	·95736	·95798	·95859	·95920	·95981
47	·96042	·96103	·96164	·96225	·96285	·96345	·96406	·96466	·96526	·96586
48	1·96646	·96706	·96766	·96826	·96885	·96944	·97003	·97062	·97121	·97181
49	·97239	·97298	·97357	·97415	·97473	·97531	·97589	·97647	·97705	·97763
50	·97821	·97879	·97937	·97994	·98051	·98108	·98165	·98222	·98279	·98336
51	1·98393	·98450	·98507	·98563	·98619	·98675	·98731	·98787	·98843	·98899
52	·98954	·99010	·99066	·99121	·99176	·99231	·99286	·99341	·99396	·99451
53	·99506	·99561	·99616	·99671	·99725	·99779	·99833	·99887	·99941	·99995
54	2·00049	·00103	·00157	·00211	·00264	·00317	·00370	·00423	·00476	·00529
55	·00582	·00635	·00688	·00740	·00793	·00845	·00898	·00950	·01003	·01055
56	·01107	·01159	·01211	·01263	·01314	·01366	·01418	·01470	·01521	·01572
57	2·01623	·01674	·01725	·01776	·01827	·01878	·01929	·01980	·02031	·02081
58	·02132	·02183	·02234	·02284	·02334	·02384	·02434	·02484	·02534	·02583
59	·02633	·02683	·02733	·02782	·02831	·02880	·02929	·02978	·03027	·03076
60	2·03125	·03174	·03223	·03271	·03320	·03368	·03417	·03465	·03514	·03562
61	·03610	·03658	·03706	·03754	·03802	·03850	·03898	·03945	·03993	·04040
62	·04088	·04135	·04183	·04230	·04277	·04324	·04371	·04418	·04465	·04512
63	2·04559	·04606	·04652	·04699	·04745	·04792	·04838	·04885	·04931	·04977
64	·05023	·05069	·05115	·05160	·05206	·05252	·05298	·05343	·05389	·05434
65	·05480	·05525	·05571	·05616	·05662	·05707	·05752	·05797	·05842	·05887
66	2·05932	·05977	·06022	·06067	·06111	·06156	·06200	·06245	·06289	·06334
67	·06378	·06422	·06466	·06510	·06554	·06598	·06642	·06686	·06729	·06773
68	·06817	·06860	·06904	·06947	·06990	·07034	·07077	·07120	·07164	·07207
69	2·07250	·07293	·07336	·07379	·07422	·07465	·07508	·07550	·07593	·07635
70	·07678	·07720	·07763	·07805	·07848	·07890	·07932	·07974	·08016	·08058
71	·08100	·08142	·08184	·08226	·08267	·08309	·08351	·08392	·08434	·08475

$\left(\frac{x}{z}\right)\phi = 90°$

β	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9
72	2·08517	·08558	·08600	·08641	·08683	·08724	·08765	·08806	·08847	·08888
73	·08929	·08970	·09011	·09051	·09092	·09133	·09173	·09214	·09254	·09295
74	·09335	·09375	·09416	·09456	·09496	·09536	·09576	·09616	·09656	·09696
75	2·09736	·09776	·09816	·09855	·09895	·09935	·09975	·10014	·10054	·10093
76	·10133	·10173	·10212	·10252	·10291	·10330	·10369	·10408	·10447	·10486
77	·10525	·10564	·10603	·10641	·10680	·10719	·10758	·10796	·10835	·10873
78	2·10912	·10950	·10989	·11027	·11066	·11104	·11142	·11180	·11218	·11256
79	·11294	·11332	·11370	·11408	·11445	·11483	·11521	·11559	·11596	·11634
80	·11672	·11710	·11747	·11785	·11822	·11860	·11897	·11934	·11972	·12009
81	2·12046	·12083	·12120	·12157	·12194	·12231	·12268	·12305	·12341	·12378
82	·12415	·12452	·12488	·12525	·12561	·12598	·12634	·12671	·12707	·12744
83	·12780	·12816	·12852	·12889	·12925	·12961	·12997	·13033	·13070	·13106
84	2·13142	·13178	·13214	·13250	·13285	·13321	·13357	·13393	·13429	·13465
85	·13500	·13536	·13571	·13607	·13642	·13677	·13712	·13748	·13783	·13818
86	·13853	·13888	·13923	·13958	·13993	·14028	·14063	·14098	·14133	·14168
87	2·14203	·14237	·14272	·14307	·14341	·14376	·14411	·14445	·14480	·14514
88	·14549	·14583	·14618	·14652	·14687	·14721	·14755	·14790	·14824	·14858
89	·14892	·14926	·14960	·14994	·15028	·15062	·15096	·15130	·15164	·15197
90	2·15231	·15265	·15298	·15332	·15366	·15399	·15433	·15466	·15500	·15533
91	·15567	·15600	·15633	·15667	·15700	·15733	·15766	·15800	·15833	·15866
92	·15899	·15932	·15965	·15998	·16031	·16064	·16097	·16129	·16162	·16195
93	2·16228	·16260	·16293	·16326	·16358	·16391	·16424	·16456	·16489	·16521
94	·16554	·16586	·16619	·16651	·16684	·16716	·16748	·16780	·16813	·16845
95	·16877	·16909	·16941	·16973	·17005	·17037	·17069	·17101	·17132	·17164
96	2·17196	·17227	·17259	·17291	·17322	·17354	·17386	·17417	·17449	·17480
97	·17512	·17543	·17575	·17606	·17638	·17669	·17701	·17732	·17763	·17795
98	·17826	·17857	·17888	·17919	·17950	·17981	·18012	·18043	·18074	·18105
99	2·18136	·18167	·18197	·18228	·18259	·18290	·18320	·18351	·18382	·18412
100	·18443									

$\beta=$	0·125		0·25		0·50		0·75		1·0	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08715	·00380	·08714	·00380	·08711	·00380	·08709	·00380	·08707	·00380
10	·17357	·01518	·17348	·01517	·17332	·01515	·17316	·01513	·17300	·01511
15	·25855	·03402	·25827	·03397	·25774	·03386	·25720	·03375	·25668	·03365
20	·34139	·06014	·34076	·05997	·33952	·05964	·33830	·05932	·33711	·05900
25	·42141	·09328	·42022	·09288	·41790	·09210	·41564	·09134	·41344	·09060
30	·49798	·13314	·49600	·13232	·49217	·13075	·48849	·12925	·48495	·12781
35	·57049	·17933	·56749	·17786	·56173	·17506	·55628	·17242	·55109	·16993
40	·63838	·23142	·63413	·22899	·62608	·22441	·61854	·22017	·61146	·21623
45	·70115	·28893	·69545	·28517	·68478	·27819	·67493	·27183	·66579	·26599
50	·75834	·35133	·75104	·34582	·73752	·33573	·72522	·32669	·71394	·31851
55	·80955	·41807	·80055	·41033	·78407	·39637	·76928	·38408	·75587	·37312
60	·85445	·48855	·84371	·47807	·82427	·45946	·80706	·44335	·79161	·42919
65	·89278	·56216	·88033	·54840	·85807	·52436	·83859	·50389	·82127	·48613
70	·92430	·63826	·91027	·62067	·88545	·59042	·86396	·56511	·84502	·54342
75	·94889	·71621	·93348	·69425	·90647	·65707	·88332	·62646	·86305	·60056
80	·96644	·79537	·94995	·76850	·92126	·72372	·89685	·68744	·87559	·65708
85	·97694	·87508	·95974	·84281	·92998	·78984	·90478	·74756	·88291	·71259
90	·98042	·95471	·96297	·91656	·93283	·85491	·90736	·80641	·88529	·76671
95	·97698	1·03363	·95981	·98920	·93007	·91845	·90488	·86358	·88302	·81909
100	·96677	1·11121	·95047	1·06015	·92197	·98002	·89763	·91870	·87640	·86944
105	·95001	1·18686	·93524	1·12889	·90886	1·03920	·88595	·97145	·86576	·91747
110	·92695	1·26000	·91443	1·19492	·89109	1·09562	·87018	1·02151	·85144	·96293
115	·89793	1·33008	·88841	1·25777	·86902	1·14892	·85067	1·06863	·83377	1·00561
120	·86333	1·39656	·85759	1·31699	·84306	1·19880	·82782	1·11255	·81312	1·04531
125	·82360	1·45895	·82243	1·37220	·81366	1·24498	·80201	1·15309	·78983	1·08189
130	·77923	1·51678	·78345	1·42301	·78127	1·28722	·77365	1·19006	·76428	1·11520
135	·73082	1·56962	·74122	1·46912	·74637	1·32531	·74318	1·22333	·73686	1·14514
140	·67902	1·61710	·69635	1·51026	·70949	1·35912	·71103	1·25280	·70794	1·17165
145	·62460	1·65888	·64953	1·54620	·67117	1·38855	·67765	1·27843	·67793	1·19469
150	·56842	1·69469	·60151	1·57681	·63197	1·41354	·64350	1·30020	·64720	1·21428
155	·51150	1·72434	55310	1·60203	·59246	1·43412	·60904	1·31815	·61615	1·23045
160	·45497	1·74778	·50514	1·62192	·55321	1·45039	·57472	1·33238	·58516	1·24330
165	·40013	1·76511	·45850	1·63665	·51480	1·46252	·54096	1·34304	·55458	1·25296
170	·34830	1·77663	·41402	1·64653	·47773	1·47076	·50818	1·35032	·52476	1·25958
175	·30080	1·78298	·37245	1·65203	·44247	1·47542	·47672	1·35448	·49599	1·26338
180	·25864	1·78487	·33439	1·65372	·40941	1·47688	·44690	1·35579	·46853	1·26459

II

$\beta=$	1·5		2·0		2·5		3·0		3·5	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08703	·00380	·08699	·00379	·08695	·00379	·08691	·00379	·08687	·00379
10	·17268	·01507	·17236	·01502	·17204	·01498	·17173	·01494	·17142	·01490
15	·25564	·03344	·25462	·03324	·25363	·03305	·25265	·03286	·25170	·03268
20	·33479	·05838	·33255	·05779	·33039	·05723	·32830	·05668	·32628	·05615
25	·40923	·08920	·40523	·08787	·40143	·08662	·39780	·08543	·39434	·08430
30	·47826	·12511	·47203	·12262	·46619	·12030	·46071	·11814	·45553	·11613
35	·54144	·16533	·53260	·16117	·52446	·15739	·51691	·15391	·50988	·15071
40	·59848	·20908	·58681	·20274	·57621	·19707	·56651	·19194	·55756	·18726
45	·64928	·25560	·63469	·24658	·62161	·23863	·60978	·23155	·59898	·22517
50	·69385	·30420	·67636	·29203	·66089	·28146	·64703	·27216	·63448	·26387
55	·73229	·35427	·71206	·33851	·69435	·32503	·67862	·31330	·66448	·30294
60	·76478	·40522	·74203	·38552	·72231	·36888	·70492	·35454	·68939	·34199
65	·79152	·45655	·76656	·43261	·74510	·41262	·72629	·39556	·70956	·38073
70	·81277	·50780	·78596	·47939	·76305	·45592	·74308	·43604	·72539	·41886
75	·82879	·55857	·80052	·52552	·77649	·49847	·75561	·47573	·73718	·45622
80	·83987	·60850	·81055	·57070	·78572	·54004	·76420	·51442	·74523	·49255
85	·84630	·65724	·81635	·61466	·79104	·58038	·76915	·55191	·74989	·52771
90	·84838	·70453	·81822	·65717	·79275	·61931	·77074	·58803	·75137	·56154
95	·84640	·75009	·81645	·69802	·79113	·65665	·76924	·62262	·74998	·59391
100	·84067	·79371	·81133	·73702	·78646	·69224	·76491	·65556	·74593	·62472
105	·83150	·83516	·80314	·77401	·77899	·72596	·75801	·68674	·73948	·65386
110	·81917	·87427	·79216	·80886	·76900	·75768	·74878	·71605	·73086	·68122
115	·80402	·91089	·77868	·84143	·75674	·78730	·73745	·74340	·72027	·70674
120	·78634	·94487	·76297	·87162	·74246	·81474	·72428	·76873	·70799	·73039
125	·76644	·97612	·74531	·89935	·72642	·83994	·70948	·79198	·69417	·75209
130	·74465	1·00453	·72598	·92456	·70886	·86283	·69328	·81310	·67906	·77182
135	·72128	1·03006	·70525	·94720	·69004	·88339	·67590	·83207	·66284	·78949
140	·69663	1·05264	·68339	·96723	·67018	·90159	·65758	·84887	·64574	·80519
145	·67105	1·07229	·66068	·98467	·64954	·91744	·63851	·86350	·62792	·81884
150	·64482	1·08901	·63738	·99952	·62834	·93095	·61893	·87599	·60963	·83050
155	·61826	1·10284	·61375	1·01183	·60681	·94217	·59901	·88636	·59101	·84020
160	·59167	1·11387	·59003	1·02166	·58518	·95113	·57898	·89467	·57223	·84798
165	·56532	1·12218	·56647	1·02910	·56364	·95793	·55901	·90097	·55353	·85390
170	·53949	1·12792	·54329	1·03425	·54240	·96265	·53927	·90535	·53500	·85802
175	·51439	1·13123	·52069	1·03723	·52164	·96538	·51994	·90790	·51684	·86041
180	·49026	1·13229	·49885	1·03819	·50151	·96630	·50116	·90872	·50914	·86117

II

$\beta=$	4·0		4·5		5·0		5·5		6·0	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08683	·00378	·08679	·00378	·08675	·00378	·08671	·00378	·08667	·00377
10	·17112	·01486	·17082	·01482	·17052	·01478	·17022	·01474	·16992	·01471
15	·25076	·03249	·24984	·03231	·24894	·03213	·24806	·03196	·24719	·03179
20	·32432	·05564	·32241	·05515	·32057	·05468	·31878	·05422	·31704	·05377
25	·39102	·08323	·38784	·08221	·38479	·08123	·38186	·08030	·37903	07940
30	·45064	·11423	·44600	·11244	·44159	·11075	·43738	·10916	·43336	·10764
35	·50329	·14773	·49710	·14495	·49127	·14236	·48576	·13993	·48053	·13764
40	·54928	·18298	·54156	·17903	·53434	·17537	·52755	·17196	·52116	·16878
45	·58905	·21938	·57986	·21409	·57133	·20922	·56335	·20472	·55588	·20055
50	·62302	·25642	·61249	·24967	·60275	·24349	·59370	·23781	·58525	·23257
55	·65165	·29370	·63992	·28538	·62912	·27781	·61911	·27089	·60981	·26454
60	·67536	·33087	·66258	·32091	·65086	·31191	·64005	·30371	·63001	·29621
65	·69453	·36766	·68089	·35602	·66840	·34555	·65689	·33606	·64625	·32739
70	·70953	·40383	·69518	·39050	·68208	·37854	·67004	·36773	·65891	·35790
75	·72069	·43919	·70580	·42414	·69223	·41071	·67978	·39860	·66829	·38761
80	·72832	·47355	·71306	·45682	·69917	·44192	·68643	·42853	·67468	·41640
85	·73271	·50676	·71722	·48837	·70314	·47203	·69024	·45737	·67835	·44414
90	·73411	·53869	·71856	·51868	·70441	·50095	·69145	·48508	·67952	·47076
95	·73279	·56922	·71730	·54766	·70322	·52858	·69032	·51153	·67842	·49617
100	·72898	·59825	·71369	·57518	·69977	·55482	·68701	·53665	·67525	·52030
105	·72290	·62569	·70792	·60119	·69428	·57960	·68176	·56036	·67021	·54307
110	·71478	·65147	·70023	·62563	·68695	·60287	·67475	·58262	·66348	·56444
115	·70483	·67550	·69081	·64842	·67798	·62457	·66616	·60337	·65523	·58436
120	·69326	·69775	·67984	·66948	·66753	·64464	·65617	·62258	·64564	·60280
125	·68026	·71817	·66753	·68882	·65581	·66306	·64496	·64020	·63487	·61971
130	·66604	·73672	·65405	·70639	·64297	·67979	·63269	·65620	·62309	·63508
135	·65078	·75338	·63960	·72218	·62920	·69483	·61950	·67059	·61043	·64889
140	·63467	·76814	·62433	·73616	·61466	·70816	·60559	·68334	·59707	·66114
145	·61790	·78101	·60844	·74837	·59951	·71979	·59109	·69448	·58314	·67184
150	·60065	·79201	·59207	·75881	·58391	·72973	·57616	·70399	·56879	·68098
155	·58310	·80115	·57541	·76748	·56802	·73801	·56094	·71192	·55416	·68860
160	·56540	·80849	·55861	·77446	·55198	·74466	·54555	·71828	·53937	·69473
165	·54772	·81407	·54180	·77974	·53593	·74972	·53017	·72315	·52457	·69940
170	·53021	·81796	·52515	·78345	·52001	·75326	·51489	·72655	·50985	·70267
175	·51300	·82022	·50875	·78561	·50433	·75532	·49983	·72853	·49535	·70458
180	·49623	·82096	·49278	·78632	·48902	·75600	·48511	·72917	·48115	·70520

$\beta=$	6·5		7·0		7·5		8·0		8·5	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08663	·00377	·08659	·00377	·08655	·00377	·08651	·00376	·08647	·00376
10	·16963	·01467	·16934	·01463	·16906	·01459	·16877	·01456	·16849	·01452
15	·24634	·03163	·24550	·03147	·24467	·03131	·24386	·03115	·24306	·03100
20	·31534	·05334	·31369	·05292	·31208	·05251	·31051	·05211	·30897	·05172
25	·37630	·07854	·37367	·07771	·37112	·07692	·36866	·07615	·36628	·07541
30	·42951	·10620	·42583	·10482	·42229	·10351	·41889	·10225	·41562	·10105
35	·47556	·13548	·47083	·13344	·46631	·13150	·46199	·12966	·45785	·12790
40	·51511	·16579	·50939	·16299	·50395	·16034	·49877	·15784	·49382	·15546
45	·54884	·19666	·54220	·19302	·53591	·18960	·52995	·18638	·52428	·18334
50	·57733	·22771	·56988	·22318	·56285	·21994	·55619	·21497	·54987	·21123
55	·60111	·25868	·59295	·25322	·58526	·24813	·57801	·24338	·57115	·23893
60	·62065	·28930	·61189	·28291	·60366	·27697	·59590	·27143	·58857	·26624
65	·63635	·31943	·62710	·31209	·61842	·30528	·61024	·29895	·60252	·29303
70	·64857	·34890	·63892	·34061	·62989	·33293	·62139	·32581	·61337	·31917
75	·65762	·37756	·64768	·36834	·63837	·35983	·62963	·35192	·62139	·34455
80	·66379	·40534	·65364	·39519	·64415	·38583	·63524	·37716	·62685	·36911
85	·66733	·43209	·65706	·42105	·64745	·41088	·63844	·40147	·62995	·39273
90	·66846	·45774	·65815	·44584	·64851	·43488	·63947	·42476	·63096	·41536
95	·66738	·48222	·65712	·46948	·64753	·45777	·63851	·44696	·63002	·43693
100	·66434	·50547	·65418	·49193	·64467	·47950	·63574	·46804	·62732	·45742
105	·65949	·52740	·64949	·51311	·64013	·50001	·63134	·48792	·62306	·47673
110	·65301	·54799	·64323	·53299	·63407	·51923	·62546	·50657	·61734	·49485
115	·64506	·56717	·63556	·55151	·62665	·53716	·61827	·52395	·61036	·51173
120	·63582	·58492	·62664	·56865	·61802	·55375	·60990	·54003	·60223	·52735
125	·62545	·60121	·61663	·58437	·60834	·56896	·60051	·55479	·59311	·54169
130	·61410	·61601	·60567	·59866	·59773	·58279	·59022	·56820	·58311	·55472
135	·60192	·62930	·59390	·61151	·58633	·59522	·57917	·58027	·57238	·56646
140	·58904	·64111	·58147	·62291	·57430	·60627	·56750	·59097	·56104	·57684
145	·57563	·65142	·56851	·63286	·56175	·61590	·55532	·60032	·54920	·58593
150	·56179	·66023	·55514	·64138	·54880	·62415	·54276	·60832	·53699	·59371
155	·54769	·66758	·54150	·64848	·53558	·63103	·52993	·61500	·52451	·60020
160	·53342	·67349	·52771	·65420	·52222	·63657	·51696	·62038	·51190	·60544
165	·51914	·67800	·51389	·65856	·50882	·64080	·50394	·62449	·49924	·60944
170	·50492	·68115	·50014	·66161	·49549	·64376	·49099	·62736	·48662	·61223
175	·49091	·68300	·48657	·66340	·48233	·64548	·47819	·62905	·47416	·61389
180	·47719	·68360	·47328	·66398	·46942	·64605	·46564	·62960	·46193	·61443

$\beta=$	9·0		9·5		10·0		10·5		11·0	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08643	·00376	·08639	·00376	·08635	·00375	·08631	·00375	·08627	·00375
10	·16821	·01448	·16793	·01445	·16766	·01441	·16739	·01438	·16712	·01434
15	·24228	·03085	·24151	·03070	·24075	·03056	·24000	·03042	·23927	·03028
20	·30748	·05135	·30603	·05099	·30460	·05063	·30320	·05028	·30184	·04994
25	·36397	·07469	·36173	·07400	·35955	·07333	·35743	·07269	·35537	·07206
30	·41246	·09989	·40941	·09878	·40647	·09771	·40362	·09668	·40087	·09569
35	·45388	·12623	·45006	·12463	·44639	·12310	·44285	·12163	·43944	·12022
40	·48909	·15321	·48457	·15106	·48023	·14902	·47606	·14707	·47205	·14520
45	·51887	·18046	·51371	·17773	·50877	·17514	·50404	·17267	·49949	·17032
50	·54387	·20771	·53815	·20438	·53269	·20121	·52747	·19820	·52246	·19534
55	·56463	·23474	·55843	·23078	·55252	·22703	·54688	·22348	·54148	·22011
60	·58162	·26137	·57502	·25678	·56874	·25245	·56275	·24835	·55702	·24446
65	·59522	·28748	·58829	·28226	·58171	·27734	·57543	·27269	·56944	·26829
70	·60579	·31295	·59860	·30711	·59178	·30161	·58528	·29642	·57908	·29151
75	·61360	·33767	·60622	·33122	·59921	·32516	·59254	·31945	·58618	·31404
80	·61892	·36158	·61141	·35453	·60427	·34791	·59749	·34167	·59101	·33578
85	·62194	·38458	·61435	·37694	·60715	·36979	·60030	·36306	·59377	·35670
90	·62291	·40661	·61530	·39842	·60808	·39074	·60121	·38352	·59466	·37671
95	·62200	·42760	·61441	·41888	·60721	·41071	·60036	·40303	·59383	·39579
100	·61938	·44753	·61186	·43830	·60472	·42965	·59793	·42153	·59145	·41388
105	·61523	·46632	·60781	·45661	·60077	·44752	·59407	·43899	·58768	·43095
110	·60967	·48396	·60239	·47380	·59549	·46428	·58892	·45535	·58264	·44694
115	·60288	·50038	·59578	·48979	·58903	·47989	·58260	·47060	·57646	·46186
120	·59497	·51558	·58807	·50461	·58151	·49434	·57525	·48471	·56928	·47565
125	·58609	·52953	·57942	·51820	·57307	·50760	·56701	·49767	·56122	·48832
130	·57636	·54221	·56994	·53055	·56382	·51966	·55798	·50944	·55239	·49984
135	·56592	·55363	·55977	·54167	·55389	·53050	·54827	·52003	·54290	·51019
140	·55488	·56373	·54900	·55153	·54339	·54013	·53801	·52944	·53286	·51940
145	·54336	·57258	·53778	·56015	·53243	·54854	·52731	·53767	·52239	·52744
150	·53147	·58015	·52618	·56753	·52111	·55575	·51624	·54471	·51157	·53433
155	·51932	·58648	·51434	·57371	·50955	·56178	·50494	·55060	·50051	·54010
160	·50703	·59158	·50235	·57868	·49784	·56663	·49350	·55535	·48931	·54474
165	·49470	·59547	·49032	·58247	·48609	·57034	·48201	·55897	·47806	·54829
170	·48240	·59820	·47831	·58514	·47436	·57294	·47053	·56152	·46683	·55078
175	·47025	·59982	·46645	·58671	·46277	·57447	·45920	·56301	·45573	·55224
180	·45832	·60034	·45480	·58722	·45138	·57497	·44804	·56350	·44480	·55272

$\beta=$	11·5		12·0		12·5		13·0		13·5	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08623	·00375	·08619	·00374	·08615	·00374	·08611	·00374	·08607	·00374
10	·16685	·01431	·16658	·01427	·16632	·01424	·16606	·01421	·16580	·01417
15	·23855	·03014	·23784	·03000	·23714	·02987	·23645	·02974	·23577	·02961
20	·30051	·04961	·29921	·04929	·29794	·04898	·29669	·04867	·29546	·04837
25	·35337	·07145	·35142	·07086	·34952	·07029	·34767	·06973	·34586	·06919
30	·39820	·09474	·39562	·09382	·39311	·09293	·39067	·09207	·38829	·09123
35	·43615	·11887	·43296	·11757	·42988	·11631	·42689	·11510	·42400	·11393
40	·46819	·14341	·46446	·14170	·46086	·14005	·45738	·13847	·45402	·13694
45	·49512	·16807	·49093	·16592	·48689	·16386	·48298	·16188	·47921	·15998
50	·51765	·19262	·51304	·19002	·50861	·18753	·50433	·18515	·50020	·18286
55	·53630	·21690	·53134	·21384	·52657	·21092	·52198	·20813	·51756	·20546
60	·55153	·24077	·54628	·23725	·54123	·23390	·53638	·23070	·53171	·22764
65	·56371	·26412	·55821	·26015	·55293	·25637	·54787	·25276	·54300	·24931
70	·57315	·28686	·56746	·28244	·56200	·27824	·55677	·27423	·55174	·27040
75	·58010	·30892	·57428	·30406	·56870	·29944	·56335	·29504	·55820	·29083
80	·58483	·33020	·57892	·32492	·57326	·31990	·56782	·31512	·56259	·31056
85	·58753	·35068	·58157	·34498	·57586	·33957	·57037	·33442	·56510	·32951
90	·58840	·37027	·58241	·36418	·57667	·35839	·57117	·35289	·56589	·34765
95	·58759	·38895	·58162	·38248	·57591	·37634	·57042	·37050	·56514	·36494
100	·58526	·40666	·57934	·39982	·57366	·39335	·56822	·38717	·56299	·38131
105	·58157	·42336	·57572	·41618	·57012	·40938	·56474	·40291	·55957	·39675
110	·57664	·43901	·57089	·43152	·56538	·42443	·56009	·41768	·55501	·41126
115	·57059	·45362	·56497	·44582	·55958	·43843	·55440	·43142	·54942	·42475
120	·56357	·46711	·55809	·45904	·55283	·45140	·54778	·44414	·54292	·43723
125	·55568	·47951	·55036	·47119	·54525	·46331	·54034	·45583	·53561	·44871
130	·54703	·49079	·54189	·48223	·53695	·47413	·53220	·46644	·52762	·45913
135	·53774	·50091	·53279	·49216	·52802	·48388	·52344	·47601	·51902	·46853
140	·52791	·50993	·52316	·50099	·51859	·49252	·51418	·48449	·50993	·47686
145	·51766	·51780	·51311	·50870	·50873	·50008	·50450	·49191	·50042	·48415
150	·50707	·52455	·50273	·51532	·49855	·50658	·49451	·49829	·49061	·49041
155	·49624	·53020	·49212	·52085	·48814	·51200	·48429	·50361	·48057	·49564
160	·48527	·53475	·48136	·52531	·47758	·51638	·47393	·50791	·47039	·49986
165	·47424	·53823	·47055	·52872	·46698	·51972	·46351	·51119	·46015	·50308
170	·46324	·54066	·45976	·53111	·45638	·52207	·45311	·51350	·44993	·50535
175	·45236	·54210	·44908	·53252	·44589	·52346	·44280	·51486	·43979	·50669
180	·44164	·54256	·43857	·53298	·43558	·52391	·43267	·51531	·42983	·50713

β=	14·0		14·5		15·0		15·5		16·0	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08604	·00373	·08600	·00373	·08596	·00373	·08592	·00373	·08588	·00372
10	·16554	·01414	·16529	·01411	·16503	·01408	·16478	·01404	·16453	·01401
15	·23509	·02949	·23443	·02936	·23377	·02924	·23313	·02912	·23249	·02900
20	·29426	·04808	·29308	·04779	·29192	·04751	·29078	·04723	·28967	·04696
25	·34410	·06866	·34237	·06815	·34069	·06765	·33905	·06716	·33744	·06669
30	·38598	·09042	·38374	·08963	·38155	·08887	·37942	·08813	·37734	·08741
35	·42119	·11281	·41847	·11172	·41582	·11067	·41325	·10964	·41074	·10865
40	·45077	·13547	·44762	·13405	·44456	·13268	·44159	·13135	·43871	·13007
45	·47556	·15815	·47203	·15639	·46861	·15469	·46530	·15305	·46209	·15147
50	·49622	·18067	·49237	·17856	·48865	·17653	·48504	·17457	·48155	·17268
55	·51329	·20289	·50917	·20042	·50519	·19805	·50134	·19577	·49761	·19358
60	·52721	·22470	·52287	·22188	·51867	·21918	·51461	·21658	·51069	·21408
65	·53831	·24601	·53379	·24285	·52942	·23981	·52520	·23689	·52112	·23409
70	·54691	·26674	·54225	·26323	·53775	·25987	·53340	·25664	·52919	·25354
75	·55325	·28682	·54848	·28298	·54387	·27930	·53942	·27577	·53513	·27238
80	·55756	·30620	·55271	·30203	·54804	·29804	·54353	·29422	·53917	·29055
85	·56002	·32483	·55513	·32036	·55042	·31607	·54588	·31195	·54148	·30801
90	·56080	·34265	·55590	·33787	·55118	·33330	·54662	·32892	·54221	·32471
95	·56007	·35963	·55518	·35456	·55047	·34971	·54592	·34506	·54152	·34061
100	·55795	·37572	·55310	·37037	·54842	·36527	·54391	·36038	·53954	·35569
105	·55459	·39089	·54980	·38529	·54517	·37994	·54070	·37482	·53639	·36991
110	·55011	·40513	·54539	·39928	·54084	·39370	·53645	·38836	·53220	·38324
115	·54462	·41839	·53999	·41232	·53553	·40652	·53122	·40097	·52705	·39566
120	·53824	·43066	·53372	·42439	·52936	·41839	·52515	·41265	·52107	·40716
125	·53106	·44193	·52666	·43546	·52242	·42928	·51832	·42337	·51435	·41771
130	·52321	·45217	·51896	·44553	·51484	·43918	·51085	·43312	·50699	·42732
135	·51476	·46140	·51064	·45460	·50666	·44811	·50280	·44191	·49907	·43596
140	·50582	·46959	·50185	·46266	·49801	·45604	·49429	·44971	·49069	·44365
145	·49648	·47676	·49267	·46971	·48898	·46298	·48540	·45654	·48193	·45037
150	·48684	·48291	·48318	·47576	·47964	·46893	·47621	·46240	·47288	·45614
155	·47697	·48805	·47348	·48081	·47009	·47390	·46680	·46729	·46361	·46096
160	·46696	·49220	·46363	·48489	·46040	·47792	·45726	·47125	·45421	·46486
165	·45689	·49537	·45372	·48801	·45065	·48099	·44766	·47427	·44475	·46784
170	·44684	·49760	·44383	·49021	·44091	·48315	·43807	·47640	·43530	·46994
175	·43687	·49892	·43403	·49151	·43126	·48443	·42856	·47766	·42593	·47118
180	·42707	·49935	·42438	·49193	·42176	·48484	·41920	·47807	·41670	·47158

$\beta=$	16·5		17·0		17·5		18·0		18·5	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08584	·00372	·08580	·00372	·08577	·00372	·08573	·00371	·08569	·00371
10	·16429	·01398	·16404	·01395	·16380	·01392	·16356	·01389	·16332	·01386
15	·23186	·02888	·23124	·02877	·23063	·02865	·23002	·02854	·22942	·02843
20	·28857	·04669	·28750	·04643	·28645	·04618	·28541	·04593	·28439	·04569
25	·33587	·06622	·33433	·06577	·33282	·06533	·33134	·06490	·32989	·06448
30	·37531	·08671	·37333	·08603	·37140	·08537	·36951	·08472	·36767	·08409
35	·40830	·10769	·40592	·10675	·40361	·10584	·40135	·10496	·39914	·10410
40	·43591	·12882	·43318	·12762	·43052	·12646	·42794	·12533	·42542	·12423
45	·45898	·14994	·45595	·14846	·45301	·14703	·45014	·14564	·44735	·14430
50	·47816	·17086	·47487	·16910	·47168	·16740	·46858	·16575	·46556	·16415
55	·49400	·19146	·49050	·18942	·48710	·18745	·48379	·18554	·48057	·18369
60	·50689	·21167	·50321	·20934	·49963	·20709	·49616	·20493	·49279	·20284
65	·51717	·23139	·51334	·22879	·50962	·22628	·50602	·22385	·50252	·22151
70	·52511	·25056	·52117	·24769	·51735	·24492	·51364	·24224	·51004	·23966
75	·53098	·26913	·52696	·26599	·52306	·26296	·51927	·26004	·51559	·25723
80	·53495	·28702	·53087	·28363	·52691	·28036	·52308	·27720	·51936	·27415
85	·53722	·30422	·53311	·30058	·52913	·29707	·52526	·29368	·52150	·29041
90	·53795	·32067	·53382	·31678	·52982	·31304	·52595	·30944	·52219	·30596
95	·53727	·33634	·53315	·33223	·52916	·32827	·52529	·32446	·52153	·32078
100	·53532	·35118	·53123	·34685	·52727	·34269	·52343	·33868	·51970	·33481
105	·53222	·36520	·52818	·36067	·52426	·35631	·52046	·35211	·51677	·34806
110	·52809	·37833	·52411	·37361	·52025	·36907	·51650	·36470	·51286	·36047
115	·52301	·39057	·51910	·38567	·51531	·38096	·51164	·37642	·50807	·37205
120	·51712	·40189	·51330	·39683	·50959	·39196	·50599	·38727	·50250	·38275
125	·51051	·41228	·50679	·40707	·50317	·40206	·49966	·39723	·49625	·39258
130	·50325	·42176	·49962	·41641	·49610	·41126	·49269	·40631	·48937	·40153
135	·49545	·43025	·49194	·42478	·48853	·41952	·48522	·41446	·48200	·40958
140	·48720	·43783	·48380	·43225	·48050	·42689	·47730	·42172	·47419	·41674
145	·47856	·44446	·47529	·43878	·47212	·43331	·46903	·42807	·46603	·42302
150	·46964	·45013	·46650	·44437	·46344	·43884	·46047	·43351	·45758	·42838
155	·46051	·45489	·45750	·44907	·45456	·44347	·45171	·43808	·44893	·43289
160	·45124	·45873	·44836	·45285	·44555	·44720	·44282	·44176	·44016	·43653
165	·44192	·46167	·43916	·45575	·43648	·45006	·43386	·44459	·43131	·43931
170	·43261	·46375	·42998	·45780	·42742	·45208	·42492	·44658	·42248	·44129
175	·42337	·46496	·42087	·45900	·41843	·45327	·41604	·44775	·41371	·44244
180	·41426	·46536	·41188	·45939	·40956	·45365	·40729	·44813	·40507	·44281

$\beta=$	19·0		19·5		20·0		21·0		22·0	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08565	·00371	·08562	·00371	·08558	·00370	·08550	·00370	·08543	·00369
10	·16308	·01383	·16284	·01380	·16261	·01377	·16214	·01371	·16169	·01365
15	·22883	·02832	·22825	·02821	·22768	·02811	·22655	·02790	·22545	·02770
20	·28339	·04545	·28240	·04521	·28143	·04498	·27953	·04453	·27769	·04410
25	·32847	·06407	·32707	·06367	·32571	·06327	·32306	·06250	·32050	·06177
30	·36586	·08348	·36409	·08288	·36236	·08230	·35901	·08118	·35579	·08010
35	·39699	·10327	·39489	·10246	·39283	·10167	·38885	·10016	·38505	·09871
40	·42297	·12317	·42058	·12214	·41824	·12113	·41373	·11919	·40942	·11736
45	·44464	·14299	·44199	·14172	·43941	·14050	·43443	·13814	·42969	·13591
50	·46262	·16260	·45977	·16110	·45698	·15965	·45161	·15686	·44650	·15423
55	·47745	·18190	·47441	·18017	·47145	·17849	·46575	·17528	·46033	·17224
60	·48951	·20081	·48632	·19885	·48322	·19694	·47725	·19330	·47158	·18987
65	·49912	·21924	·49581	·21705	·49259	·21492	·48640	·21086	·48053	·20704
70	·50654	·23716	·50314	·23474	·49984	·23240	·49349	·22793	·48746	·22373
75	·51202	·25450	·50855	·25186	·50518	·24931	·49870	·24445	·49255	·23987
80	·51574	·27121	·51222	·26836	·50880	·26561	·50223	·26037	·49600	·25543
85	·51785	·28725	·51431	·28420	·51087	·28126	·50426	·27565	·49798	·27037
90	·51854	·30261	·51499	·29937	·51153	·29623	·50489	·29026	·49860	·28465
95	·51789	·31723	·51435	·31380	·51091	·31049	·50430	·30418	·49802	·29825
100	·51608	·33108	·51256	·32748	·50914	·32400	·50257	·31738	·49633	·31115
105	·51319	·34415	·50970	·34038	·50632	·33674	·49982	·32981	·49364	·32330
110	·50933	·35640	·50589	·35248	·50255	·34869	·49613	·34147	·49004	·33470
115	·50460	·36783	·50123	·36376	·49795	·35982	·49164	·35233	·48565	·34531
120	·49910	·37839	·49580	·37418	·49258	·37012	·48640	·36239	·48053	·35514
125	·49293	·38810	·48970	·38377	·48656	·37959	·48052	·37163	·47478	·36418
130	·48614	·39693	·48300	·39249	·47995	·38820	·47407	·38005	·46848	·37240
135	·47887	·40488	·47582	·40034	·47285	·39596	·46714	·38762	·46170	·37980
140	·47116	·41195	·46821	·40732	·46533	·40285	·45979	·39435	·45452	·38639
145	·46310	·41814	·46024	·41343	·45746	·40889	·45210	·40025	·44700	·39215
150	·45476	·42344	·45201	·41867	·44933	·41407	·44417	·40532	·43924	·39711
155	·44622	·42789	·44358	·42307	·44100	·41841	·43602	·40955	·43128	·40125
160	·43756	·43148	·43502	·42661	·43254	·42191	·42775	·41299	·42319	·40460
165	·42882	·43423	·42639	·42933	·42402	·42460	·41944	·41560	·41506	·40717
170	·42010	·43618	·41777	·43125	·41550	·42649	·41111	·41744	·40690	·40897
175	·41143	·43732	·40921	·43238	·40704	·42761	·40284	·41854	·39881	·41004
180	·40290	·43769	·40078	·43275	·39870	·42797	·39468	·41889	·39082	·41039

II

$\beta=$	23		24		25		26		27	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08535	·00369	·08528	·00368	·08521	·00368	·08513	·00367	·08506	·00367
10	·16123	·01359	·16079	·01354	·16035	·01348	·15992	·01343	·15949	·01338
15	·22438	·02750	·22333	·02731	·22230	·02712	·22130	·02694	·22032	·02676
20	·27591	·04368	·27418	·04327	·27250	·04288	·27087	·04250	·26928	·04213
25	·31803	·06107	·31564	·06039	·31333	·05974	·31110	·05911	·30894	·05850
30	·35269	·07907	·34971	·07808	·34684	·07713	·34407	·07622	·34140	·07535
35	·38141	·09733	·37791	·09601	·37455	·09475	·37131	·09354	·36818	·09238
40	·40529	·11561	·40134	·11395	·39755	·11236	·39390	·11084	·39039	·10939
45	·42516	·13379	·42082	·13177	·41666	·12985	·41267	·12802	·40883	·12627
50	·44162	·15173	·43696	·14936	·43249	·14711	·42821	·14496	·42410	·14291
55	·45516	·16937	·45023	·16664	·44551	·16405	·44099	·16159	·43665	·15924
60	·46618	·18662	·46102	·18355	·45609	·18063	·45137	·17786	·44684	·17522
65	·47493	·20343	·46960	·20002	·46451	·19678	·45963	·19370	·45495	·19077
70	·48171	·21976	·47624	·21601	·47101	·21246	·46601	·20908	·46121	·20586
75	·48670	·23556	·48112	·23148	·47579	·22761	·47070	·22395	·46582	·22047
80	·49008	·25078	·48443	·24638	·47905	·24221	·47389	·23826	·46895	·23452
85	·49201	·26540	·48632	·26070	·48089	·25626	·47570	·25204	·47073	·24803
90	·49262	·27937	·48692	·27438	·48148	·26966	·47628	·26519	·47130	·26094
95	·49205	·29268	·48636	·28741	·48092	·28243	·47573	·27771	·47076	·27323
100	·49039	·30529	·48474	·29976	·47934	·29453	·47418	·28958	·46924	·28488
105	·48776	·31718	·48216	·31140	·47682	·30594	·47170	·30077	·46680	·29586
110	·48424	·32833	·47872	·32232	·47345	·31665	·46840	·31127	·46356	·30616
115	·47995	·33871	·47451	·33249	·46931	·32662	·46434	·32105	·45958	·31577
120	·47494	·34833	·46961	·34191	·46452	·33585	·45964	·33010	·45496	·32465
125	·46932	·35718	·46410	·35057	·45911	·34433	·45434	·33842	·44976	·33282
130	·46315	·36521	·45806	·35844	·45319	·35204	·44853	·34599	·44406	·34025
135	·45651	·37246	·45156	·36554	·44682	·35901	·44228	·35282	·43793	·34695
140	·44949	·37891	·44468	·37185	·44008	·36519	·43567	·35888	·43143	·35290
145	·44214	·38454	·43748	·37737	·43302	·37061	·42874	·36420	·42463	·35812
150	·43453	·38939	·43003	·38212	·42571	·37526	·42157	·36876	·41759	·36260
155	·42675	·39345	·42240	·38610	·41823	·37915	·41423	·37258	·41039	·36635
160	·41883	·39673	·41464	·38931	·41062	·38231	·40676	·37568	·40305	·36940
165	·41086	·39925	·40683	·39178	·40296	·38472	·39924	·37805	·39566	·37173
170	·40287	·40101	·39900	·39351	·39528	·38643	·39170	·37973	·38825	·37337
175	·39494	·40206	·39123	·39454	·38766	·38744	·38422	·38072	·38090	·37434
180	·38712	·40240	·38356	·39487	·38014	·38776	·37683	·38103	·37364	·37465

II

$\beta=$	28		29		30		31		32	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08499	·00366	·08491	·00366	·08484	·00366	·08477	·00365	·08470	·00365
10	·15907	·01332	·15865	·01327	·15824	·01322	·15784	·01317	·15744	·01312
15	·21935	·02659	·21841	·02642	·21748	·02626	·21657	·02610	·21568	·02594
20	·26773	·04177	·26623	·04143	·26476	·04109	·26332	·04076	·26192	·04044
25	·30684	·05792	·30481	·05736	·30283	·05681	·30090	·05628	·29903	·05576
30	·33881	·07451	·33631	·07370	·33388	·07292	·33153	·07217	·32924	·07144
35	·36517	·09127	·36226	·09020	·35944	·08917	·35671	·08818	·35408	·08723
40	·38701	·10800	·38376	·10666	·38061	·10538	·37756	·10415	·37462	·10296
45	·40514	·12459	·40159	·12298	·39816	·12144	·39485	·11996	·39165	·11854
50	·42015	·14095	·41634	·13907	·41268	·13727	·40914	·13555	·40573	·13389
55	·43248	·15700	·42847	·15485	·42461	·15280	·42088	·15083	·41729	·14894
60	·44249	·17269	·43831	·17028	·43429	·16797	·43041	·16576	·42667	·16364
65	·45046	·18797	·44613	·18529	·44199	·18274	·43799	·18029	·43413	·17795
70	·45661	·20280	·45219	·19987	·44794	·19708	·44384	·19440	·43989	·19184
75	·46114	·21714	·45664	·21396	·45231	·21093	·44815	·20803	·44413	·20526
80	·46421	·23095	·45966	·22754	·45528	·22429	·45107	·22118	·44701	·21820
85	·46596	·24422	·46138	·24059	·45698	·23711	·45274	·23379	·44865	·23061
90	·46652	·25690	·46193	·25305	·45752	·24937	·45327	·24585	·44917	·24248
95	·46599	·26897	·46142	·26491	·45701	·26103	·45277	·25732	·44868	·25377
100	·46449	·28041	·45993	·27616	·45555	·27209	·45133	·26820	·44727	·26448
105	·46210	·29119	·45759	·28674	·45325	·28250	·44907	·27845	·44504	·27457
110	·45892	·30131	·45446	·29669	·45017	·29228	·44604	·28807	·44206	·28404
115	·45501	·31074	·45061	·30595	·44638	·30139	·44231	·29703	·43840	·29286
120	·45047	·31947	·44615	·31453	·44200	·30983	·43800	·30534	·43415	·30104
125	·44537	·32749	·44115	·32242	·43708	·31758	·43316	·31296	·42937	·30854
130	·43977	·33479	·43564	·32960	·43166	·32464	·42783	·31991	·42413	·31538
135	·43374	·34137	·42972	·33605	·42584	·33099	·42210	·32616	·41849	·32154
140	·42736	·34722	·42345	·34182	·41967	·33666	·41603	·33173	·41251	·32702
145	·42068	·35235	·41687	·34686	·41320	·34162	·40966	·33662	·40625	·33183
150	·41376	·35675	·41008	·35118	·40652	·34587	·40309	·34080	·39977	·33596
155	·40668	·36044	·40311	·35482	·39966	·34945	·39634	·34432	·39312	·33942
160	·39947	·36343	·39602	·35776	·39269	·35234	·38947	·34717	·38636	·34222
165	·39221	·36572	·38888	·36000	·38566	·35455	·38255	·34935	·37955	·34437
170	·38493	·36734	·38172	·36160	·37862	·35612	·37562	·35089	·37272	·34588
175	·37770	·36829	·37461	·36253	·37162	·35704	·36873	·35179	·36592	·34678
180	·37056	·36860	·36759	·36284	·36471	·35735	·36192	·35210	·35921	·34708

$\beta=$	33		34		35		36		37	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08463	·00364	·08456	·00364	·08449	·00363	·08442	·00363	·08435	·00362
10	·15704	·01307	·15665	·01302	·15627	·01297	·15589	·01293	·15551	·01288
15	·21481	·02578	·21395	·02563	·21311	·02548	·21229	·02534	·21148	·02520
20	·26055	·04012	·25921	·03982	·25791	·03953	·25663	·03924	·25538	·03896
25	·29721	·05526	·29544	·05478	·29371	·05431	·29203	·05385	·29039	·05341
30	·32702	·07073	·32486	·07005	·32276	·06939	·32073	·06875	·31875	·06813
35	·35152	·08631	·34904	·08542	·34663	·08456	·34429	·08372	·34201	·08291
40	·37177	·10182	·36901	·10071	·36633	·09964	·36373	·09861	·36120	·09761
45	·38855	·11717	·38556	·11585	·38266	·11458	·37984	·11335	·37710	·11216
50	·40243	·13229	·39924	·13076	·39615	·12928	·39315	·12785	·39024	·12647
55	·41382	·14712	·41046	·14537	·40721	·14368	·40406	·14206	·40101	·14049
60	·42306	·16160	·41956	·15964	·41618	·15775	·41291	·15594	·40974	·15419
65	·43040	·17570	·42680	·17354	·42332	·17146	·41995	·16945	·41668	·16752
70	·43607	·18938	·43239	·18702	·42883	·18474	·42539	·18255	·42205	·18044
75	·44025	·20260	·43651	·20005	·43289	·19759	·42939	·19522	·42600	·19294
80	·44309	·21534	·43931	·21260	·43565	·20996	·43211	·20742	·42868	·20497
85	·44470	·22757	·44089	·22464	·43721	·22183	·43365	·21912	·43019	·21651
90	·44522	·23925	·44140	·23615	·43771	·23317	·43414	·23031	·43068	·22756
95	·44474	·25037	·44093	·24710	·43725	·24397	·43368	·24095	·43023	·23805
100	·44335	·26091	·43956	·25749	·43590	·25421	·43235	·25105	·42892	·24801
105	·44116	·27086	·43740	·26729	·43377	·26387	·43025	·26058	·42685	·25741
110	·43821	·28018	·43450	·27648	·43091	·27293	·42744	·26951	·42407	·26622
115	·43461	·28886	·43095	·28503	·42741	·28135	·42399	·27782	·42067	·27442
120	·43043	·29692	·42683	·29297	·42335	·28918	·41998	·28554	·41672	·28204
125	·42571	·30432	·42218	·30026	·41877	·29636	·41546	·29262	·41226	·28903
130	·42056	·31105	·41710	·30689	·41376	·30290	·41052	·29907	·40738	·29538
135	·41501	·31712	·41163	·31288	·40836	·30881	·40520	·30489	·40214	·30112
140	·40912	·32250	·40583	·31818	·40265	·31403	·39956	·31005	·39656	·30622
145	·40295	·32724	·39975	·32285	·39665	·31864	·39365	·31459	·39074	·31070
150	·39656	·33133	·39346	·32688	·39045	·32261	·38753	·31850	·38469	·31455
155	·39000	·33473	·38699	·33023	·38407	·32592	·38124	·32177	·37849	·31778
160	·38335	·33749	·38043	·33295	·37760	·32860	·37486	·32441	·37219	·32038
165	·37664	·33961	·37382	·33504	·37108	·33066	·36842	·32644	·36584	·32239
170	·36991	·34108	·36718	·33649	·36453	·33209	·36196	·32786	·35946	·32380
175	·36320	·34198	·36056	·33738	·35800	·33296	·35552	·32872	·35311	·32465
180	·35658	·34228	·35404	·33767	·35157	·33324	·34917	·32900	·34683	·32492

$\beta=$	38		39		40		41		42	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08428	·00362	·08421	·00362	·08414	·00361	·08407	·00361	·08400	·00360
10	·15514	·01283	·15477	·01279	·15441	·01274	·15405	·01270	·15369	·01266
15	·21069	·02506	·20991	·02492	·20914	·02478	·20838	·02464	·20764	·02451
20	·25416	·03869	·25297	·03842	·25180	·03816	·25065	·03790	·24953	·03765
25	·28879	·05298	·28723	·05256	·28570	·05215	·28421	·05175	·28275	·05136
30	·31682	·06753	·31494	·06694	·31310	·06637	·31131	·06582	·30956	·06528
35	·33980	·08213	·33764	·08137	·33554	·08063	·33349	·07991	·33150	·07922
40	·35875	·09665	·35637	·09571	·35405	·09481	·35179	·09393	·34959	·09308
45	·37445	·11101	·37187	·10990	·36936	·10882	·36691	·10777	·36454	·10676
50	·38742	·12514	·38468	·12385	·38201	·12260	·37941	·12139	·37689	·12022
55	·39805	·13898	·39517	·13751	·39238	·13610	·38966	·13473	·38702	·13340
60	·40667	·15250	·40369	·15086	·40079	·14928	·39797	·14775	·39523	·14627
65	·41352	·16565	·41045	·16384	·40747	·16210	·40457	·16041	·40175	·15878
70	·41882	·17841	·41568	·17644	·41263	·17454	·40967	·17270	·40679	·17092
75	·42271	·19074	·41952	·18861	·41643	·18656	·41342	·18458	·41050	·18266
80	·42536	·20261	·42213	·20033	·41900	·19813	·41596	·19601	·41300	·19395
85	·42685	·21400	·42361	·21158	·42046	·20924	·41739	·20698	·41442	·20479
90	·42733	·22490	·42408	·22233	·42093	·21986	·41787	·21747	·41489	·21516
95	·42688	·23525	·42364	·23256	·42049	·22996	·41743	·22745	·41446	·22502
100	·42559	·24508	·42237	·24226	·41924	·23954	·41621	·23691	·41325	·23437
105	·42355	·25436	·42035	·25142	·41724	·24858	·41422	·24583	·41129	·24318
110	·42081	·26305	·41764	·26000	·41457	·25705	·41158	·25420	·40868	·25145
115	·41745	·27115	·41433	·26799	·41130	·26495	·40835	·26201	·40549	·25917
120	·41355	·27866	·41048	·27540	·40749	·27226	·40459	·26923	·40177	·26630
125	·40915	·28556	·40614	·28222	·40321	·27899	·40036	·27587	·39760	·27286
130	·40434	·29183	·40139	·28841	·39852	·28511	·39573	·28192	·39302	·27884
135	·39916	·29750	·39627	·29400	·39346	·29063	·39073	·28738	·38808	·28423
140	·39366	·30253	·39084	·29897	·38810	·29554	·38543	·29223	·38284	·28903
145	·38791	·30695	·38516	·30334	·38249	·29985	·37989	·29648	·37736	·29323
150	·38194	·31075	·37927	·30709	·37667	·30356	·37414	·30015	·37168	·29685
155	·37582	·31394	·37323	·31023	·37071	·30666	·36825	·30321	·36586	·29988
160	·36960	·31651	·36708	·31278	·36464	·30918	·36226	·30570	·35994	·30234
165	·36333	·31849	·36088	·31473	·35851	·31111	·35620	·30761	·35395	·30423
170	·35703	·31989	·35467	·31611	·35237	·31247	·35013	·30895	·34795	·30556
175	·35076	·32072	·34847	·31693	·34625	·31328	·34408	·30975	·34197	·30634
180	·34456	·32099	·34235	·31720	·34020	·31354	·33811	·31001	·33606	·30660

$\beta=$	43		44		45		46		47	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08393	·00360	·08387	·00359	·08380	·00359	·08373	·00358	·08366	·00358
10	·15334	·01261	·15299	·01257	·15265	·01253	·15231	·01249	·15197	·01245
15	·20691	·02438	·20619	·02426	·20548	·02414	·20478	·02402	·20409	·02390
20	·24843	·03741	·24735	·03717	·24629	·03694	·24525	·03671	·24423	·03649
25	·28132	·05098	·27992	·05061	·27855	·05024	·27721	·04989	·27590	·04955
30	·30785	·06475	·30618	·06424	·30455	·06374	·30296	·06326	·30140	·06279
35	·32955	·07855	·32765	·07789	·32579	·07725	·32398	·07663	·32221	·07602
40	·34744	·09225	·34535	·09144	·34330	·09066	·34131	·08990	·33936	·08916
45	·36223	·10578	·35997	·10482	·35777	·10389	·35562	·10299	·35353	·10211
50	·37443	·11908	·37204	·11798	·36971	·11691	·36744	·11587	·36522	·11485
55	·38445	·13211	·38194	·13086	·37950	·12965	·37712	·12847	·37480	·12732
60	·39256	·14483	·38997	·14344	·38744	·14209	·38497	·14078	·38256	·13950
65	·39901	·15719	·39634	·15566	·39374	·15418	·39121	·15274	·38874	·15133
70	·40399	·16920	·40126	·16753	·39861	·16591	·39602	·16434	·39350	·16281
75	·40766	·18080	·40489	·17900	·40220	·17725	·39957	·17556	·39701	·17391
80	·41013	·19196	·40733	·19003	·40460	·18816	·40195	·18634	·39937	·18458
85	·41153	·20268	·40872	·20063	·40598	·19864	·40332	·19671	·40072	·19484
90	·41199	·21292	·40917	·21075	·40643	·20865	·40376	·20661	·40116	·20463
95	·41157	·22267	·40876	·22039	·40603	·21818	·40336	·21604	·40076	·21396
100	·41037	·23191	·40757	·22952	·40485	·22721	·40219	·22497	·39960	·22279
105	·40844	·24061	·40566	·23813	·40295	·23572	·40032	·23339	·39776	·23113
110	·40586	·24879	·40311	·24621	·40044	·24371	·39783	·24128	·39529	·23894
115	·40270	·25641	·39999	·25375	·39734	·25117	·39477	·24867	·39227	·24624
120	·39903	·26346	·39636	·26072	·39376	·25807	·39123	·25549	·38876	·25299
125	·39491	·26995	·39229	·26713	·38973	·26440	·38724	·26175	·38481	·25919
130	·39038	·27586	·38781	·27298	·38530	·27019	·38286	·26748	·38048	·26485
135	·38550	·28119	·38298	·27824	·38053	·27539	·37814	·27262	·37581	·26994
140	·38032	·28593	·37786	·28293	·37546	·28002	·37312	·27721	·37084	·27448
145	·37490	·29008	·37250	·28704	·37016	·28409	·36788	·28123	·36566	·27846
150	·36929	·29366	·36695	·29058	·36467	·28760	·36245	·28470	·36028	·28189
155	·36353	·29666	·36126	·29354	·35904	·29052	·35688	·28760	·35477	·28476
160	·35768	·29909	·35547	·29595	·35332	·29290	·35122	·28995	·34916	·28709
165	·35175	·30096	·34961	·29780	·34752	·29473	·34548	·29176	·34349	·28888
170	·34582	·30228	·34374	·29910	·34172	·29602	·33974	·29303	·33780	·29014
175	·33991	·30305	·33790	·29986	·33593	·29677	·33401	·29378	·33213	·29088
180	·33406	·30330	·33211	·30011	·33021	·29702	·32835	·29403	·32653	·29113

$\beta=$	48		49		50		52		54	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08360	·00358	·08353	·00357	·08347	·00357	·08333	·00356	·08320	·00355
10	·15164	·01241	·15131	·01237	·15099	·01233	·15035	·01225	·14972	·01217
15	·20342	·02378	·20276	·02366	·20210	·02355	·20082	·02334	·19957	·02313
20	·24323	·03627	·24225	·03605	·24128	·03584	·23940	·03543	·23758	·03504
25	·27461	·04921	·27335	·04888	·27211	·04856	·26971	·04794	·26740	·04734
30	·29987	·06232	·29837	·06187	·29691	·06143	·29406	·06058	·29133	·05976
35	·32048	·07543	·31879	·07485	·31713	·07429	·31392	·07320	·31084	·07216
40	·33746	·08843	·33560	·08773	·33378	·08704	·33026	·08571	·32689	·08445
45	·35148	·10126	·34948	·10043	·34753	·09962	·34375	·09805	·34013	·09656
50	·36305	·11387	·36094	·11291	·35887	·11198	·35487	·11018	·35105	·10847
55	·37254	·12621	·37034	·12513	·36818	·12407	·36400	·12204	·36000	·12011
60	·38022	·13826	·37793	·13705	·37570	·13588	·37138	·13363	·36725	·13148
65	·38633	·14997	·38398	·14865	·38169	·14736	·37725	·14488	·37301	·14253
70	·39104	·16133	·38865	·15989	·38630	·15849	·38178	·15580	·37745	·15324
75	·39451	·17231	·39207	·17075	·38969	·16924	·38510	·16634	·38071	·16359
80	·39685	·18287	·39439	·18121	·39199	·17959	·38736	·17649	·38293	·17355
85	·39819	·19302	·39572	·19126	·39331	·18954	·38865	·18624	·38419	·18311
90	·39862	·20271	·39614	·20084	·39373	·19903	·38906	·19555	·38459	·19225
95	·39822	·21194	·39575	·20998	·39333	·20808	·38867	·20442	·38422	·20095
100	·39707	·22068	·39460	·21862	·39220	·21663	·38756	·21281	.38313	·20919
105	·39525	·22893	·39280	·22679	·39041	·22472	·38581	·22074	·38141	·21697
110	·39281	·23667	·39039	·23445	·38803	·23230	·38347	·22817	·37911	·22426
115	·38982	·24388	·38743	·24159	·38510	·23937	·38060	·23510	·37630	·23106
120	·38635	·25056	·38400	·24820	·38170	·24591	·37726	·24152	·37302	·23736
125	·38244	·25670	·38012	·25428	·37786	·25193	·37350	·24743	·36933	·24316
130	·37815	·26230	·37588	·25982	·37366	·25741	·36938	·25280	·36528	·24843
135	·37353	·26734	·37131	·26481	·36914	·26236	·36494	·25765	·36092	·25319
140	·36862	·27183	·36645	·26926	·36433	·26676	·36023	·26197	·35630	·25743
145	·36349	·27577	·36137	·27316	·35930	·27062	·35530	·26575	·35147	·26114
150	·35817	·27916	·35611	·27651	·35409	·27394	·35018	·26902	·34644	·26436
155	·35271	·28201	·35070	·27934	·34873	·27674	·34492	·27176	·34127	·26705
160	·34716	·28431	·34520	·28161	·34329	·27898	·33958	·27396	·33603	·26921
165	·34154	·28608	·33964	·28337	·33778	·28073	·33418	·27567	·33073	·27089
170	·33591	·28733	·33406	·28461	·33225	·28196	·32875	·27688	·32540	·27207
175	·33030	·28807	·32851	·28534	·32675	·28269	·32335	·27760	·32009	·27278
180	·32475	·28831	·32301	·28557	·32131	·28291	·31801	·27782	·31485	·27300

β =	56		58		60		62		64	
φ	x/b	z/b	x/b	z/b	x/b	z/b	x/b	z/b	x/b	z/b
5°	·08307	·00354	·08294	·00353	·08281	·00352	·08268	·00352	·08256	·00351
10	·14910	·01210	·14849	·01203	·14790	·01196	·14732	·01189	·14675	·01182
15	·19836	·02292	·19718	·02272	·19603	·02252	·19491	·02233	·19382	·02215
20	·23582	·03466	·23411	·03429	·23246	·03394	·23086	·03360	·22930	·03327
25	·26516	·04677	·26300	·04622	·26092	·04569	·25891	·04518	·25696	·04468
30	·28870	·05898	·28617	·05823	·28372	·05751	·28136	·05682	·27908	·05616
35	·30787	·07117	·30502	·07022	·30227	·06931	·29963	·06844	·29707	·06760
40	·32364	·08324	·32052	·08209	·31752	·08099	·31463	·07994	·31185	·07892
45	·33665	·09515	·33331	·09380	·33010	·09250	·32701	·09126	·32404	·09007
50	·34738	·10684	·34386	·10529	·34048	·10380	·33723	·10237	·33409	·10101
55	·35617	·11828	·35250	·11653	·34897	·11485	·34558	·11325	·34231	·11172
60	·36329	·12944	·35950	·12749	·35585	·12563	·35234	·12386	·34897	·12216
65	·36894	·14029	·36505	·13816	·36131	·13612	·35773	·13418	·35427	·13232
70	·37331	·15081	·36934	·14849	·36553	·14628	·36187	·14417	·35835	·14215
75	·37651	·16097	·37249	·15848	·36863	·15610	·36492	·15383	·36135	·15166
80	·37869	·17075	·37463	·16809	·37073	·16555	·36698	·16313	·36338	·16081
85	·37993	·18014	·37585	·17731	·37193	·17462	·36816	·17205	·36454	·16959
90	·38032	·18912	·37623	·18614	·37230	·18329	·36853	·18058	·36491	·17799
95	·37995	·19766	·37586	·19453	·37194	·19154	·36818	·18869	·36456	·18597
100	·37889	·20575	·37482	·20249	·37092	·19937	·36717	·19639	·36357	·19355
105	·37720	·21339	·37316	·20998	·36929	·20674	·36557	·20365	·36199	·20069
110	·37494	·22055	·37094	·21702	·36711	·21366	·36343	·21045	·35988	·20739
115	·37218	·22723	·36823	·22359	·36444	·22012	·36079	·21681	·35729	·21364
120	·36896	·23342	·36507	·22967	·36133	·22610	·35774	·22269	·35428	·21943
125	·36534	·23911	·36151	·23526	·35783	·23159	·35429	·22809	·35089	·22475
130	·36136	·24429	·35760	·24035	·35399	·23660	·35051	·23302	·34717	·22960
135	·35708	·24897	·35339	·24495	·34985	·24112	·34644	·23747	·34316	·23398
140	·35254	·25313	·34893	·24904	·34546	·24514	·34212	·24142	·33891	·23788
145	·34779	·25678	·34426	·25263	·34086	·24867	·33759	·24490	·33445	·24130
150	·34285	·25993	·33940	·25572	·33609	·25171	·33290	·24789	·32983	·24424
155	·33778	·26257	·33442	·25831	·33119	·25426	·32808	·25040	·32508	·24671
160	·33262	·26470	·32935	·26041	·32620	·25633	·32317	·25243	·32025	·24872
165	·32741	·26635	·32422	·26203	·32116	·25792	·31821	·25400	·31536	·25026
170	·32218	·26751	·31909	·26318	·31611	·25905	·31323	·25511	·31046	·25135
175	·31697	·26820	·31397	·26385	·31107	·25971	·30827	·25576	·30557	·25199
180	·31181	·26842	·30889	·26407	·30607	·25993	·30335	·25598	·30072	·25221

β =	66		68		70		72		74	
φ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08243	·00350	·08231	·00349	·08219	·00349	·08207	·00348	·08195	·00347
10	·14619	·01175	·14564	·01169	·14510	·01162	·14457	·01156	·14405	·01150
15	·19276	·02197	·19172	·02180	·19070	·02163	·18971	·02147	·18874	·02131
20	·22779	·03295	·22632	·03264	·22488	·03234	·22349	·03205	·22213	·03177
25	·25507	·04420	·25324	·04374	·25145	·04330	·24972	·04287	·24804	·04245
30	·27688	·05552	·27474	·05491	·27267	·05432	·27066	·05374	·26871	·05318
35	·29460	·06680	·29221	·06602	·28990	·06527	·28766	·06455	·28549	·06385
40	·30917	·07794	·30657	·07700	·30406	·07610	·30162	·07523	·29926	·07439
45	·32117	·08893	·31840	·08783	·31572	·08678	·31312	·08576	·31061	·08478
50	·33107	·09970	·32815	·09845	·32533	·09724	·32260	·09608	·31996	·09496
55	·33917	·11025	·33613	·10884	·33320	·10749	·33036	·10618	·32762	·10492
60	·34572	·12053	·34259	·11897	·33956	·11747	·33664	·11603	·33381	·11464
65	·35094	·13054	·34773	·12883	·34463	·12718	·34163	·12560	·33873	·12408
70	·35495	·14021	·35168	·13836	·34852	·13658	·34547	·13487	·34252	·13322
75	·35791	·14958	·35459	·14758	·35139	·14567	·34830	·14383	·34531	·14206
80	·35991	·15860	·35656	·15647	·35333	·15442	·35022	·15246	·34721	·15057
85	·36105	·16723	·35769	·16498	·35444	·16282	·35131	·16074	·34828	·15874
90	·36142	·17551	·35805	·17313	·35480	·17085	·35166	·16865	·34862	·16654
95	·36107	·18336	·35771	·18087	·35446	·17848	·35133	·17618	·34830	·17397
100	·36010	·19083	·35675	·18822	·35352	·18572	·35040	·18332	·34738	·18101
105	·35854	·19786	·35522	·19515	·35201	·19255	·34891	·19005	·34591	·18765
110	·35646	·20446	·35317	·20165	·34999	·19896	·34692	·19637	·34395	·19388
115	·35391	·21061	·35066	·20771	·34752	·20493	·34448	·20226	·34154	·19970
120	·35095	·21631	·34774	·21333	·34464	·21047	·34164	·20772	·33874	·20508
125	·34761	·22155	·34445	·21849	·34139	·21556	·33844	·21274	·33558	·21004
130	·34395	·22633	·34084	·22320	·33783	·22019	·33493	·21731	·33212	·21454
135	·34000	·23064	·33695	·22745	·33400	·22439	·33115	·22144	·32839	·21861
140	·33581	·23449	·33282	·23124	·32993	·22812	·32714	·22512	·32444	·22224
145	·33142	·23786	·32849	·23456	·32566	·23139	·32293	·22835	·32029	·22543
150	·32687	·24075	·32401	·23741	·32125	·23421	·31857	·23113	·31598	·22817
155	·32219	·24318	·31940	·23981	·31670	·23657	·31409	·23346	·31156	·23047
160	·31743	·24517	·31471	·24176	·31207	·23849	·30952	·23535	·30705	·23234
165	·31261	·24668	·30996	·24325	·30739	·23996	·30491	·23681	·30250	·23378
170	·30778	·24775	·30519	·24431	·30269	·24101	·30027	·23784	·29792	·23479
175	·30296	·24839	·30044	·24494	·29801	·24163	·29565	·23845	·29337	·23539
180	·29818	·24861	·29573	·24515	·29336	·24183	·29107	·23865	·28885	·23559

$\beta=$	76		78		80		82		84	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08183	·00346	·08171	·00346	·08159	·00345	·08147	·00344	·08136	·00343
10	·14353	·01144	·14303	·01138	·14253	·01133	·14204	·01126	·14156	·01121
15	·18780	·02115	·18688	·02100	·18597	·02085	·18508	·02070	·18421	·02056
20	·22081	·03149	·21952	·03123	·21826	·03097	·21703	·03072	·21583	·03047
25	·24640	·04205	·24481	·04166	·24326	·04128	·24175	·04091	·24028	·04055
30	·26681	·05265	·26497	·05213	·26318	·05162	·26144	·05113	·25974	·05066
35	·28338	·06318	·28134	·06252	·27935	·06189	·27742	·06127	·27554	·06068
40	·29697	·07358	·29476	·07280	·29260	·07204	·29051	·07131	·28847	·07060
45	·30817	·08383	·30581	·08291	·30352	·08203	·30129	·08118	·29913	·08035
50	·31740	·09388	·31492	·09284	·31251	·09183	·31018	·09086	·30791	·08992
55	·32496	·10371	·32238	·10254	·31988	·10141	·31746	·10032	·31510	·09927
60	·33107	·11330	·32842	·11201	·32584	·11076	·32334	·10955	·32091	·10838
65	·33592	·12261	·33320	·12120	·33057	·11984	·32801	·11853	·32553	·11725
70	·33967	·13164	·33690	·13011	·33422	·12863	·33162	·12720	·32909	·12582
75	·34242	·14036	·33962	·13872	·33690	·13713	·33427	·13559	·33171	·13411
80	·34429	·14875	·34146	·14700	·33872	·14531	·33606	·14368	·33348	·14210
85	·34535	·15681	·34251	·15495	·33976	·15316	·33709	·15143	·33450	·14976
90	·34569	·16451	·34285	·16256	·34009	·16067	·33741	·15885	·33482	·15709
95	·34537	·17184	·34253	·16979	·33978	·16781	·33711	·16590	·33452	·16406
100	·34446	·17879	·34163	·17665	·33889	·17458	·33623	·17258	·33365	·17066
105	·34301	·18534	·34019	·18311	·33747	·18097	·33483	·17890	·33227	·17690
110	·34107	·19149	·33829	·18918	·33559	·18696	·33297	·18481	·33043	·18274
115	·33870	·19723	·33594	·19485	·33327	·19255	·33068	·19033	·32817	·18819
120	·33593	·20254	·33321	·20009	·33058	·19773	·32802	·19545	·32554	·19325
125	·33282	·20743	·33015	·20491	·32755	·20249	·32503	·20015	·32258	·19790
130	·32940	·21188	·32676	·20931	·32421	·20684	·32174	·20445	·31933	·20214
135	·32572	·21589	·32314	·21327	·32063	·21075	·31820	·20831	·31583	·20596
140	·32182	·21947	·31928	·21680	·31682	·21424	·31443	·21177	·31211	·20938
145	·31772	·22262	·31523	·21991	·31282	·21731	·31048	·21480	·30821	·21237
150	·31347	·22533	·31104	·22259	·30868	·21995	·30639	·21740	·30416	·21494
155	·30911	·22760	·30673	·22484	·30443	·22217	·30219	·21960	·30001	·21711
160	·30466	·22944	·30234	·22665	·30009	·22396	·29790	·22136	·29577	·21886
165	·30017	·23087	·29790	·22806	·29570	·22535	·29356	·22273	·29148	·22021
170	·29565	·23186	·29344	·22904	·29130	·22632	·28921	·22369	·28719	·22116
175	·29115	·23245	·28899	·22962	·28690	·22690	·28487	·22428	·28289	·22174
180	·28669	·23265	·28459	·22982	·28255	·22709	·28057	·22446	·27864	·22192

$\beta=$	86		88		90		92		94	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08124	·00343	·08113	·00342	·08101	·00341	·08090	·00340	·08079	·00340
10	·14109	·01115	·14062	·01110	·14016	·01104	·13971	·01099	·13926	·01094
15	·18336	·02042	·18253	·02029	·18172	·02016	·18092	·02003	·18013	·01990
20	·21466	·03023	·21352	·03000	·21240	·02977	·21131	·02955	·21024	·02933
25	·23885	·04021	·23745	·03987	·23608	·03954	·23475	·03922	·23345	·03891
30	·25809	·05020	·25648	·04975	·25491	·04931	·25338	·04889	·25188	·04848
35	·27371	·06010	·27192	·05955	·27018	·05901	·26849	·05849	·26684	·05798
40	·28649	·06991	·28456	·06924	·28268	·06859	·28085	·06796	·27907	·06735
45	·29703	·07955	·29498	·07877	·29299	·07802	·29105	·07729	·28916	·07658
50	·30571	·08901	·30356	·08812	·30147	·08726	·29944	·08643	·29746	·08562
55	·31281	·09825	·31058	·09726	·30842	·09630	·30631	·09537	·30425	·09446
60	·31855	·10725	·31626	·10616	·31403	·10510	·31186	·10408	·30975	·10308
65	·32312	·11601	·32077	·11482	·31849	·11366	·31627	·11254	·31411	·11145
70	·32663	·12449	·32425	·12320	·32193	·12195	·31967	·12074	·31747	·11956
75	·32923	·13268	·32681	·13130	·32446	·12997	·32217	·12867	·31995	·12741
80	·33097	·14057	·32854	·13910	·32617	·13767	·32387	·13629	·32163	·13494
85	·33198	·14815	·32953	·14659	·32715	·14508	·32484	·14361	·32258	·14218
90	·33230	·15539	·32985	·15374	·32747	·15214	·32515	·15060	·32289	·14910
95	·33200	·16228	·32955	·16055	·32716	·15888	·32485	·15726	·32260	·15569
100	·33114	·16880	·32870	·16700	·32633	·16526	·32403	·16357	·32179	·16193
105	·32978	·17496	·32736	·17309	·32501	·17127	·32272	·16952	·32049	·16782
110	·32796	·18074	·32556	·17880	·32322	·17693	·32095	·17511	·31874	·17335
115	·32573	·18612	·32335	·18413	·32104	·18220	·31879	·18033	·31660	·17851
120	·32313	·19112	·32078	·18907	·31850	·18708	·31628	·18516	·31412	·18329
125	·32020	·19572	·31789	·19362	·31564	·19158	·31345	·18960	·31132	·18768
130	·31699	·19991	·31471	·19776	·31249	·19567	·31034	·19365	·30824	·19169
135	·31353	·20369	·31129	·20149	·30911	·19936	·30699	·19730	·30493	·19530
140	·30986	·20707	·30766	·20483	·30552	·20267	·30344	·20057	·30141	·19853
145	·30600	·21002	·30385	·20775	·30176	·20555	·29972	·20342	·29773	·20136
150	·30199	·21256	·29989	·21027	·29784	·20805	·29584	·20589	·29389	·20380
155	·29789	·21471	·29583	·21238	·29383	·21013	·29187	·20795	·28996	·20584
160	·29370	·21644	·29169	·21410	·28973	·21183	·28782	·20964	·28596	·20751
165	·28946	·21777	·28750	·21542	·28558	·21314	·28372	·21093	·28190	·20879
170	·28522	·21871	·28330	·21635	·28142	·21407	·27960	·21185	·27782	·20970
175	·28097	·21928	·27910	·21691	·27728	·21461	·27550	·21239	·27376	·21023
180	·27676	·21946	·27494	·21709	·27316	·21479	·27143	·21256	·26973	·21040

$\beta=$	96		97		98		99		100	
ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
5°	·08068	·00339	·08062	·00339	·08057	·00339	·08051	·00338	·08046	·00338
10	·13882	·01089	·13860	·01086	·13839	·01084	·13817	·01081	·13796	·01079
15	·17936	·01978	·17898	·01972	·17860	·01966	·17823	·01960	·17786	·01954
20	·20919	·02912	·20867	·02901	·20816	·02891	·20765	·02881	·20715	·02871
25	·23217	·03860	·23154	·03845	·23092	·03830	·23031	·03815	·22970	·03801
30	·25042	·04808	·24970	·04788	·24899	·04769	·24829	·04749	·24760	·04730
35	·26522	·05748	·26443	·05723	·26364	·05699	·26287	·05675	·26210	·05652
40	·27733	·06676	·27648	·06647	·27564	·06618	·27481	·06589	·27398	·06561
45	·28731	·07589	·28640	·07555	·28551	·07522	·28463	·07489	·28376	·07456
50	·29553	·08483	·29458	·08445	·29364	·08407	·29271	·08370	·29180	·08333
55	·30225	·09358	·30127	·09315	·30030	·09273	·29934	·09231	·29839	·09190
60	·30769	·10211	·30668	·10164	·30568	·10117	·30469	·10071	·30372	·10025
65	·31200	·11039	·31097	·10988	·30995	·10937	·30894	·10887	·30795	·10837
70	·31533	·11842	·31428	·11786	·31324	·11731	·31222	·11677	·31121	·11623
75	·31778	·12618	·31672	·12558	·31567	·12499	·31463	·12441	·31361	·12383
80	·31944	·13364	·31837	·13301	·31731	·13238	·31626	·13176	·31523	·13115
85	·32039	·14080	·31931	·14012	·31825	·13946	·31720	·13880	·31616	·13816
90	·32069	·14765	·31961	·14694	·31855	·14624	·31750	·14555	·31646	·14487
95	·32041	·15417	·31933	·15342	·31827	·15269	·31722	·15196	·31618	·15125
100	·31960	·16035	·31853	·15957	·31747	·15881	·31642	·15805	·31539	·15731
105	·31831	·16617	·31724	·16536	·31619	·16457	·31515	·16379	·31412	·16302
110	·31658	·17164	·31552	·17080	·31448	·16998	·31345	·16917	·31243	·16838
115	·31447	·17675	·31342	·17589	·31239	·17504	·31137	·17420	·31036	·17338
120	·31201	·18148	·31098	·18059	·30996	·17972	·30895	·17886	·30795	·17801
125	·30924	·18582	·30822	·18491	·30722	·18402	·30622	·18314	·30524	·18227
130	·30620	·18979	·30519	·18886	·30420	·18795	·30322	·18705	·30226	·18616
135	·30292	·19337	·30193	·19242	·30096	·19149	·30000	·19057	·29905	·18966
140	·29944	·19656	·29847	·19560	·29752	·19465	·29657	·19371	·29564	·19279
145	·29579	·19936	·29484	·19838	·29390	·19742	·29297	·19647	·29206	·19554
150	·29200	·20177	·29107	·20078	·29015	·19980	·28924	·19884	·28835	·19790
155	·28811	·20380	·28720	·20280	·28630	·20182	·28541	·20085	·28453	·19989
160	·28414	·20545	·28325	·20444	·28237	·20344	·28150	·20246	·28064	·20150
165	·28013	·20671	·27926	·20569	·27840	·20469	·27755	·20371	·27671	·20274
170	·27609	·20761	·27524	·20659	·27440	·20558	·27357	·20460	·27275	·20362
175	·27207	·20814	·27124	·20712	·27042	·20611	·26961	·20512	·26880	·20414
180	·26808	·20831	·26727	·20729	·26646	·20628	·26567	·20529	·26489	·20431

III

$V \div b^3$										
ϕ	0·125	0·25	0·50	0·75	1·0	1·5	2·0	2·5	3·0	4·0
5°	·00006	·00005	·00005	·00005	·00005	·00005	·00005	·00005	·00005	·00005
10	·00072	·00072	·00072	·00072	·00071	·00071	·00070	·00070	·00070	·00069
15	·00360	·00358	·00356	·00353	·00351	·00346	·00341	·00337	·00333	·00324
20	·01111	·01106	·01092	·01081	·01067	·01042	·01019	·00997	·00976	·00937
25	·02646	·02621	·02573	·02526	·02480	·02397	·02320	·02247	·02180	·02058
30	·05314	·05243	·05107	·04979	·04858	·04636	·04435	·04253	·04087	·03794
35	·09480	·09312	·08994	·08701	·08428	·07938	·07507	·07125	·06784	·06199
40	·15485	·15135	·14487	·13899	·13363	·12419	·11613	·10914	·10302	·09277
45	·23618	·22962	·21768	·20708	·19758	·18125	·16764	·15611	·14618	·12990
50	·34087	·32957	·30934	·29176	·27630	·25027	·22913	·21155	·19666	·17272
55	·47003	·45181	·41986	·39265	·36914	·33040	·29964	·27451	·25353	·22036
60	·62360	·59592	·54826	·50853	·47477	·42022	·37781	·34372	·31562	·27183
65	·80038	·76038	·69275	·63748	·59129	·51800	·46209	·41779	·38168	·32612
70	·99803	·94263	·85068	·77703	·71637	·62172	·55075	·49521	·45039	·38218
75	1·21315	1·13934	1·01898	·92431	·84743	·72928	·64202	·57450	·52048	·43903
80	1·44153	1·34646	1·19402	1·07624	·98177	·83855	·73420	·65423	·59072	·49573
85	1·67825	1·55955	1·37220	1·22968	1·11672	·94751	·82566	·73307	·66000	·55144
90	1·91816	1·77394	1·54971	1·38160	1·24972	1·05425	·91493	·80982	·72730	·60544
95	2·15591	1·98508	1·72306	1·52920	1·37848	1·15712	1·00072	·88344	·79178	·65707
100	2·38640	2·18865	1·88911	1·66998	1·50099	1·25468	1·08195	·95306	·85272	·70582
105	2·60497	2·38085	2·04507	1·80188	1·61557	1·34579	1·15773	1·01800	·90954	·75128
110	2·80760	2·55849	2·18878	1·92326	1·72095	1·42955	1·22743	1·07773	·96183	·79314
115	2·99113	2·71912	2·31860	2·03295	1·81623	1·50539	1·29059	1·13193	1·00932	·83121
120	3·15332	2·86112	2·43356	2·13024	1·90089	1·57296	1·34700	1·18041	1·05185	·86538
125	3·29302	2·98367	2·53321	2·21490	1·97477	1·63217	1·39659	1·22314	1·08941	·89565
130	3·40994	3·08677	2·61773	2·28707	2·03803	1·68318	1·43948	1·26021	1·12209	·92207
135	3·50479	3·17111	2·68767	2·34728	2·09108	1·72630	1·47593	1·29183	1·15003	·94478
140	3·57908	3·23802	2·74404	2·39631	2·13460	1·76201	1·50631	1·31831	1·17352	·96396
145	3·63503	3·28928	2·78818	2·43519	2·16942	1·79091	1·53108	1·34001	1·19284	·97983
150	3·67521	3·32703	2·82159	2·46508	2·19648	1·81367	1·55076	1·35736	1·20837	·99266
155	3·70248	3·35352	2·84588	2·48725	2·21679	1·83104	1·56591	1·37081	1·22046	1·00274
160	3·71978	3·37109	2·86270	2·50294	2·23139	1·84373	1·57712	1·38083	1·22953	1·01035
165	3·72980	3·38189	2·87361	2·51341	2·24126	1·85250	1·58495	1·38789	1·23595	1·01580
170	3·73493	3·38785	2·88002	2·51973	2·24735	1·85802	1·58994	1·39244	1·24011	1·01935
175	3·73705	3·39056	2·88315	2·52292	2·25048	1·86092	1·59261	1·39488	1·24237	1·02129
180	3·73756	3·39122	2·88398	2·52382	2·25138	1·86178	1·59340	1·39562	1·24305	1·02188

$V \div b^3$

β

ϕ	5	6	7	8	10	12	14	16	20	24
5°	·00005	·00004	·00004	·00004	·00004	·00004	·00004	·00004	·00004	·00004
10	·00068	·00067	·00066	·00066	·00064	·00063	·00062	·00060	·00058	·00057
15	·00317	·00309	·00302	·00296	·00283	·00272	·00262	·00252	·00235	·00221
20	·00901	·00868	·00838	·00810	·00760	·00716	·00677	·00643	·00584	·00535
25	·01950	·01854	·01767	·01689	·01553	·01439	·01341	·01257	·01117	·01006
30	·03543	·03326	·03136	·02968	·02683	·02451	·02257	·02093	·01828	·01624
35	·05713	·05303	·04951	·04645	·04139	·03736	·03407	·03132	·02699	·02372
40	·08449	·07764	·07187	·06694	·05892	·05266	·04763	·04349	·03706	·03229
45	·11707	·10665	·09801	·09071	·07902	·07006	·06294	·05715	·04827	·04177
50	·15422	·13945	·12736	·11725	·10126	·08917	·07967	·07200	·06037	·05195
55	·19519	·17536	·15930	·14599	·12517	·10960	·09747	·08776	·07314	·06264
60	·23912	·21364	·19319	·17637	·15030	·13097	·11604	·10414	·08635	·07367
65	·28515	·25356	·22840	·20784	·17620	·15292	·13505	·12088	·09981	·08488
70	·33245	·29442	·26433	·23988	·20246	·17513	·15424	·13775	·11334	·09613
75	·38020	·33556	·30041	·27198	·22871	·19727	·17335	·15452	·12676	·10727
80	·42769	·37635	·33614	·30373	·25460	·21908	·19214	·17100	·13993	·11819
85	·47424	·41627	·37105	·33471	·27984	·24030	·21042	·18702	·15272	·12878
90	·51926	·45485	·40474	·36460	·30416	·26074	·22801	·20243	·16502	·13897
95	·56227	·49166	·43689	·39310	·32733	·28022	·24477	·21710	·17672	·14867
100	·60286	·52640	·46722	·41998	·34919	·29859	·26057	·23095	·18777	·15782
105	·64072	·55880	·49551	·44507	·36959	·31574	·27533	·24388	·19809	·16637
110	·67560	·58867	·52161	·46822	·38844	·33159	·28898	·25584	·20764	·17429
115	·70736	·61589	·54541	·48935	·40565	·34608	·30147	·26679	·21640	·18155
120	·73592	·64041	·56686	·50841	·42121	·35919	·31277	·27672	·22435	·18815
125	·76126	·66220	·58596	·52540	·43510	·37091	·32289	·28561	·23147	·19408
130	·78345	·68131	·60274	·54034	·44734	·38126	·33184	·29348	·23780	·19934
135	·80258	·69783	·61727	·55330	·45799	·39028	·33965	·30036	·24333	·20395
140	·81879	·71187	·62965	·56436	·46710	·39801	·34636	·30627	·24810	·20794
145	·83227	·72357	·63999	·57363	·47475	·40452	·35202	·31127	·25214	·21131
150	·84321	·73311	·64845	·58121	·48104	·40989	·35669	·31540	·25549	·21412
155	·85185	·74067	·65516	·58726	·48607	·41419	·36044	·31873	·25819	·21639
160	·85841	·74643	·66030	·59189	·48994	·41751	·36335	·32131	·26029	·21816
165	·86313	·75059	·66402	·59526	·49277	·41994	·36548	·32320	·26184	·21947
170	·86623	·75334	·66649	·59750	·49466	·42157	·36691	·32448	·26289	·22036
175	·86792	·75487	·66786	·59875	·49571	·42249	·36772	·32520	·26348	·22086
180	·86846	·75534	·66829	·59913	·49604	·42277	·36798	·32543	·26367	·22102

$V \div b^3$

ϕ	28	32	40	48	56	64	72	80	88	96	100
5°	·000042	·000041	·000040	·000040	·000039	·000038	·000037	·000036	·000036	·000035	·000035
10	·000539	·000520	·000488	·000459	·000434	·000412	·000392	·000374	·000358	·000343	·000336
15	·002077	·001964	·001773	·001617	·001487	·001381	·001283	·001201	·001129	·001066	·001037
20	·004943	·004596	·004032	·003596	·003245	·002958	·002717	·002513	·002338	·002185	·002115
25	·009159	·008408	·007227	·006339	·005646	·005089	·004631	·004248	·003922	·003642	·003516
30	·014616	·013290	·011249	·009750	·008600	·007689	·006950	·006337	·005821	·005381	·005184
35	·021157	·019095	·015974	·013721	·012015	·010678	·009602	·008717	·007977	·007348	·007068
40	·028608	·025671	·021279	·018149	·015803	·013980	·012522	·011329	·010337	·009497	·009124
45	·036796	·032866	·027045	·022938	·019884	·017525	·015648	·014121	·012854	·011786	·011312
50	·045555	·040538	·033162	·027999	·024184	·021252	·018929	·017045	·015486	·014175	·013596
55	·054728	·048551	·039525	·033249	·028635	·025103	·022314	·020057	·018194	·016632	·015942
60	·064170	·056782	·046042	·038614	·033175	·029025	·025756	·023117	·020944	·019125	·018322
65	·073745	·065118	·052625	·044024	·037747	·032970	·029216	·026191	·023705	·021625	·020708
70	·083334	·073454	·059198	·049418	·042300	·036896	·032657	·029247	·026446	·024107	·023077
75	·092824	·081698	·065689	·054739	·046790	·040764	·036045	·032254	·029144	·026550	·025408
80	·102121	·089768	·072037	·059940	·051175	·044541	·039353	·035189	·031776	·028931	·027680
85	·111136	·097592	·078187	·064976	·055419	·048196	·042553	·038028	·034322	·031235	·029878
90	·119798	·105107	·084093	·069812	·059495	·051705	·045624	·040752	·036765	·033446	·031987
95	·128045	·112261	·089715	·074415	·063374	·055045	·048548	·043347	·039090	·035550	·033994
100	·135827	·119012	·095022	·078761	·067037	·058199	·051310	·045796	·041288	·037538	·035891
105	·143103	·125327	·099987	·082828	·070466	·061153	·053897	·048091	·043347	·039401	·037669
110	·149845	·131180	·104593	·086602	·073650	·063895	·056299	·050224	·045261	·041134	·039321
115	·156033	·136556	·108826	·090074	·076579	·066421	·058512	·052189	·047024	·042730	·040845
120	·161657	·141444	·112680	·093237	·079249	·068724	·060531	·053982	·048634	·044188	·042237
125	·166714	·145842	·116151	·096088	·081659	·070804	·062356	·055603	·050089	·045507	·043495
130	·171208	·149755	·119243	·098631	·083810	·072661	·063986	·057052	·051391	·046687	·044622
135	·175152	·153191	·121963	·100871	·085706	·074300	·065425	·058332	·052542	·047730	·045619
140	·178562	·156166	·124322	·102814	·087353	·075725	·066677	·059447	·053545	·048640	·046487
145	·181459	·158697	·126332	·104474	·088761	·076943	·067749	·060402	·054404	·049420	·047232
150	·183870	·160806	·128010	·105862	·089939	·077965	·068648	·061203	·055125	·050075	·047858
155	·185823	·162515	·129374	·106991	·090899	·078798	·069382	·061857	·055715	·050611	·048371
160	·187349	·163853	·130443	·107878	·091655	·079453	·069960	·062374	·056180	·051034	·048775
165	·188479	·164845	·131238	·108538	·092218	·079943	·070392	·062760	·056529	·051351	·049077
170	·189246	·165519	·131779	·108989	·092603	·080278	·070688	·063024	·056768	·051569	·049287
175	·189682	·165904	·132088	·109247	·092823	·080470	·070858	·063177	·056906	·051694	·049407
180	·189820	·166027	·132186	·109329	·092894	·080532	·070913	·063225	·056950	·051734	·049445

IV

$\beta = -0{\cdot}1$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 43 . 44	·099834	·004996	0·0001
0·2	11 . 27 . 12	·198670	·019929	0·0012
0·3	17 . 10 . 10	·295526	·044639	0·0062
0·4	22 . 52 . 22	·389443	·078864	0·0190
0·5	28 . 33 . 33	·479501	·122239	0·0451
0·6	34 . 13 . 29	·564827	·174310	0·0901
0·7	39 . 51 . 57	·644608	·234533	0·1597
0·8	45 . 28 . 42	·718100	·302289	0·2589
0·9	51 . 3 . 32	·784634	·376891	0·3917
1·0	56 . 36 . 16	·843623	·457591	0·5602
1·1	62 . 6 . 41	·894566	·543597	0·7648
1·2	67 . 34 . 37	·937052	·634081	1·0037
1·3	72 . 59 . 52	·970764	·728188	1·2733
1·4	78 . 22 . 17	·995474	·825049	1·5679
1·5	83 . 41 . 41	1·011047	·923792	1·8806
1·6	88 . 57 . 53	1·017438	1·023553	2·2035
1·7	94 . 10 . 41	1·014689	1·123480	2·5281
1·8	99 . 19 . 51	1·002922	1·222752	2·8459
1·9	104 . 25 . 8	·982342	1·320578	3·1492
2·0	109 . 26 . 10	·953225	1·416211	3·4311

$\beta = -0{\cdot}2$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 43 . 41	·099834	·004995	0·0001
0·2	11 . 26 . 52	·198671	·019924	0·0012
0·3	17 . 9 . 1	·295532	·044615	0·0062
0·4	22 . 49 . 38	·389468	·078788	0·0190
0·5	28 . 28 . 14	·479576	·122061	0·0450
0·6	34 . 4 . 21	·565010	·173956	0·0898
0·7	39 . 37 . 31	·644996	·233909	0·1592
0·8	45 . 7 . 18	·718838	·301286	0·2581
0·9	50 . 33 . 18	·785928	·375390	0·3903
1·0	55 . 55 . 5	·845749	·455478	0·5583
1·1	61 . 12 . 19	·897879	·540775	0·7624
1·2	66 . 24 . 39	·941991	·630481	1·0014
1·3	71 . 31 . 43	·977853	·723794	1·2720
1·4	76 . 33 . 13	1·005325	·819913	1·5694
1·5	81 . 28 . 48	1·024357	·918054	1·8874
1·6	86 . 18 . 10	1·034980	1·017458	2·2189
1·7	91 . 0 . 56	1·037303	1·117403	2·5565
1·8	95 . 36 . 45	1·031506	1·217208	2·8924
1·9	100 . 5 . 9	1·017835	1·316243	3·2195
2·0	104 . 25 . 35	·996592	1·413937	3·5312
2·1	108 . 37 . 25	·968134	1·509779	3·8221
2·2	112 . 39 . 46	·932865	1·603330	4·0881

$\beta = -0{\cdot}3$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 43 . 39	·099834	·004995	0·0001
0·2	11 . 26 . 31	·198672	·019919	0·0012
0·3	17 . 7 . 52	·295538	·044590	0·0062
0·4	22 . 46 . 55	·389493	·078713	0·0190
0·5	28 . 22 . 56	·479651	·121883	0·0449
0·6	33 . 55 . 15	·565192	·173602	0·0896
0·7	39 . 23 . 9	·645381	·233286	0·1587
0·8	44 . 46 . 1	·719570	·300283	0·2572
0·9	50 . 3 . 14	·787210	·373889	0·3889
1·0	55 . 14 . 13	·847851	·453361	0·5562
1·1	60 . 18 . 27	·901148	·537936	0·7599
1·2	65 . 15 . 25	·946855	·626844	0·9988
1·3	70 . 4 . 37	·984825	·719323	1·2703
1·4	74 . 45 . 38	1·015003	·814631	1·5701
1·5	79 . 17 . 59	1·037420	·912059	1·8929
1·6	83 . 41 . 13	1·052187	1·010938	2·2324
1·7	87 . 54 . 55	1·059484	1·110649	2·5820
1·8	91 . 58 . 33	1·059554	1·210628	2·9350
1·9	95 . 51 . 35	1·052696	1·310373	3·2849
2·0	99 . 33 . 25	1·039258	1·409449	3·6258
2·1	103 . 3 . 18	1·019627	1·507487	3·9526
2·2	106 . 20 . 22	·994228	1·604194	4·2609
2·3	109 . 23 . 30	·963519	1·699349	4·5477
2·4	112 . 11 . 20	·927992	1·792815	4·8106
2·5	114 . 42 . 7	·888167	1·884534	5·0484
2·6	116 . 53 . 35	·844606	1·974541	5·2609
2·7	118 . 42 . 49	·797913	2·062965	5·4484
2·8	120 . 5 . 59	·748753	2·150045	5·6122
2·9	120 . 58 . 6	·697878	2·236135	5·7538
3·0	121 . 12 . 37	·646154	2·321719	5·8753
3·1	120 . 41 . 5	·594624	2·407419	5·9790

IV

$\beta=-0{\cdot}4$					$\beta=-0{\cdot}4$				
$\frac{s}{b}$	ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$	$\frac{s}{b}$	ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
	° ′ ″					° ′ ″			
0·1	5 . 43 . 36	·099834	·004995	0·0001	4·0	79 . 26 . 59	·690340	3·344917	8·1825
0·2	11 . 26 . 11	·198673	·019914	0·0012	4·1	75 . 9 . 20	·712338	3·442443	8·3330
0·3	17 . 6 . 42	·295544	·044566	0·0062	4·2	71 . 4 . 3	·741417	3·538100	8·4915
0·4	22 . 44 . 11	·389518	·078638	0·0190	4·3	67 . 15 . 11	·777026	3·631525	8·6604
0·5	28 . 17 . 39	·479725	·121706	0·0448	4·4	63 . 44 . 45	·818532	3·722488	8·8422
0·6	33 . 46 . 10	·565374	·173249	0·0894	4·5	60 . 33 . 6	·865284	3·810872	9·0388
0·7	39 . 8 . 51	·645764	·232663	0·1583	4·6	57 . 39 . 26	·916662	3·896652	9·2525
0·8	44 . 24 . 50	·720297	·299280	0·2563	4·7	55 . 2 . 16	·972104	3·979865	9·4855
0·9	49 . 33 . 22	·788479	·372386	0·3875	4·8	52 . 39 . 43	1·031116	4·060587	9·7398
1·0	54 . 33 . 40	·849929	·451238	0·5542	4·9	50 . 29 . 49	1·093278	4·138912	10·0173
1·1	59 . 25 . 3	·904373	·535082	0·7573	5·0	48 . 30 . 40	1·158232	4·214938	10·3198
1·2	64 . 6 . 54	·951646	·623171	0·9961	5·1	46 . 40 . 30	1·225683	4·288758	10·6492
1·3	68 . 38 . 35	·991680	·714779	1·2682	5·2	44 . 57 . 42	1·295386	4·360458	11·0070
1·4	72 . 59 . 31	1·024505	·809212	1·5701	5·3	43 . 20 . 53	1·367139	4·430106	11·3947
1·5	77 . 9 . 12	1·050232	·905824	1·8972	5·4	41 . 48 . 51	1·440775	4·497760	11·8135
1·6	81 . 7 . 3	1·069048	1·004017	2·2440	5·5	40 . 20 . 32	1·516160	4·563461	12·2646
1·7	84 . 52 . 34	1·081208	1·103257	2·6047	5·6	38 . 55 . 4	1·593180	4·627237	12·7487
1·8	88 . 25 . 11	1·087023	1·203072	2·9736	5·7	37 . 31 . 43	1·671741	4·689105	13·2666
1·9	91 . 44 . 19	1·086851	1·303058	3·3450	5·8	36 . 9 . 51	1·751766	4·749069	13·8185
2·0	94 . 49 . 23	1·081094	1·402880	3·7138	5·9	34 . 48 . 58	1·833186	4·807122	14·4043
2·1	97 . 39 . 38	1·070186	1·502273	4·0754	6·0	33 . 28 . 38	1·915945	4·863251	15·0239
2·2	100 . 14 . 19	1·054592	1·601041	4·4259	6·1	32 . 8 . 32	1·999991	4·917435	15·6763
2·3	102 . 32 . 31	1·034805	1·699057	4·7622	6·2	30 . 48 . 22	2·085277	4·969645	16·3605
2·4	104 . 33 . 12	1·011338	1·796259	5·0820	6·3	29 . 27 . 57	2·171758	5·019850	17·0749
2·5	106 . 15 . 9	·984733	1·892651	5·3838	6·4	28 . 7 . 8	2·259392	5·068014	17·8175
2·6	107 . 36 . 58	·955558	1·988298	5·6668	6·5	26 . 45 . 48	2·348138	5·114098	18·5857
2·7	108 . 37 . 4	·924411	2·083323	5·9306	6·6	25 . 23 . 53	2·437953	5·158062	19·3765
2·8	109 . 13 . 38	·891926	2·177899	6·1757	6·7	24 . 1 . 22	2·528793	5·199865	20·1862
2·9	109 . 24 . 39	·858782	2·272246	6·4029	6·8	22 . 38 . 14	2·620615	5·239468	21·0107
3·0	109 . 7 . 59	·825711	2·366619	6·6132	6·9	21 . 14 . 32	2·713369	5·276832	21·8454
3·1	108 . 21 . 30	·793504	2·461290	6·8081	7·0	19 . 50 . 19	2·807009	5·311919	22·6848
3·2	107 . 3 . 13	·763016	2·556527	6·9893	7·1	18 . 25 . 40	2·901483	5·344696	23·5235
3·3	105 . 11 . 40	·735167	2·652566	7·1585	7·2	17 . 0 . 39	2·996736	5·375131	24·3548
3·4	102 . 46 . 17	·710926	2·749576	7·3177	7·3	15 . 35 . 25	3·092714	5·403198	25·1718
3·5	99 . 47 . 57	·691284	2·847616	7·4689	7·4	14 . 10 . 4	3·189358	5·428875	25·9673
3·6	96 . 19 . 25	·677195	2·946603	7·6143	7·5	12 . 44 . 45	3·286610	5·452145	26·7334
3·7	92 . 25 . 33	·669515	3·046290	7·7561	7·6	11 . 19 . 37	3·384410	5·472997	27·4620
3·8	88 . 13 . 18	·668915	3·146265	7·8965	7·7	9 . 54 . 49	3·482694	5·491426	28·1440
3·9	83 . 50 . 54	·675814	3·246002	8·0379	7·8	8 . 30 . 32	3·581403	5·507431	28·7710

IV

$\beta = -0{\cdot}4$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
7·9	7. 6.55	3·680473	5·521021	29·3334
8·0	5.44. 9	3·779842	5·532210	29·8220
8·1	4.22.26	3·879451	5·541018	30·2273
8·2	3. 1.55	3·979240	5·547473	30·5400
8·3	1.42.47	4·079153	5·551609	30·7505
8·4	0.25.14	4·179133	5·553466	30·8494

$\beta = -0{\cdot}5$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5.43.34	·099834	·004994	0·0001
0·2	11.25.50	·198673	·019909	0·0012
0·3	17. 5.33	·295550	·044541	0·0062
0·4	22.41.28	·389543	·078562	0·0189
0·5	28.12.22	·479799	·121528	0·0448
0·6	33.37. 7	·565554	·172896	0·0892
0·7	38.54.35	·646144	·232041	0·1578
0·8	44. 3.46	·721016	·298278	0·2555
0·9	49. 3.40	·789734	·370883	0·3860
1·0	53.53.24	·851981	·449110	0·5521
1·1	58.32. 8	·907553	·532214	0·7546
1·2	62.59. 6	·956362	·619464	0·9931
1·3	67.13.35	·998419	·710165	1·2657
1·4	71.14.54	1·033831	·803663	1·5693
1·5	75. 2.27	1·062789	·899359	1·9002
1·6	78.35.37	1·085557	·996716	2·2534
1·7	81.53.51	1·102461	1·095263	2·6243
1·8	84.56.35	1·113880	1·194597	3·0079
1·9	87.43.14	1·120238	1·294385	3·3993
2·0	90.13.15	1·121993	1·394362	3·7944
2·1	92.26. 0	1·119635	1·494328	4·1891
2·2	94.20.53	1·113677	1·594145	4·5803
2·3	95.57.16	1·104653	1·693734	4·9654
2·4	97.14.27	1·093117	1·793064	5·3423
2·5	98.11.45	1·079638	1·892151	5·7098
2·6	98.48.31	1·064803	1·991044	6·0671
2·7	99. 4. 6	1·049215	2·089821	6·4138
2·8	98.57.55	1·033490	2·188577	6·7503
2·9	98.29.34	1·018260	2·287410	7·0770
3·0	97.38.49	1·004169	2·386412	7·3949

$\beta = -0{\cdot}5$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
3·1	96.25.45	·991863	2·485650	7·7054
3·2	94.50.48	·981987	2·585158	8·0097
3·3	92.54.51	·975171	2·684920	8·3097
3·4	90.39.17	·972012	2·784864	8·6071
3·5	88. 5.59	·973058	2·884851	8·9040
3·6	85.17.18	·978790	2·984676	9·2024
3·7	82.15.54	·989603	3·084078	9·5046
3·8	79. 4.42	1·005790	3·182747	9·8129
3·9	75.46.36	1·027542	3·280338	10·1296
4·0	72.24.22	1·054942	3·376496	10·4567
4·1	69. 0.26	1·087976	3·470867	10·7968
4·2	65.36.52	1·126547	3·563113	11·1518
4·3	62.15.13	1·170487	3·652926	11·5237
4·4	58.56.37	1·219580	3·740030	11·9141
4·5	55.41.49	1·273575	3·824184	12·3247
4·6	52.31.11	1·332202	3·905180	12·7563
4·7	49.24.52	1·395179	3·982842	13·2097
4·8	46.22.47	1·462225	4·057021	13·6851
4·9	43.24.47	1·533062	4·127589	14·1820
5·0	40.30.37	1·607419	4·194438	14·6996
5·1	37.40. 0	1·685034	4·257477	15·2360
5·2	34.52.41	1·765652	4·316628	15·7887
5·3	32. 8.24	1·849028	4·371823	16·3548
5·4	29.26.59	1·934924	4·423010	16·9301
5·5	26.48.14	2·023109	4·470142	17·5097
5·6	24.12. 5	2·113361	4·513188	18·0878
5·7	21.38.26	2·205460	4·552125	18·6577
5·8	19. 7.18	2·299194	4·586943	19·2121
5·9	16.38.43	2·394357	4·617643	19·7428
6·0	14.12.45	2·490747	4·644239	20·2408
6·1	11.49.31	2·588172	4·666758	20·6966
6·2	9.29. 9	2·686442	4·685240	21·0999
6·3	7.11.50	2·785378	4·699739	21·4403
6·4	4.57.44	2·884810	4·710322	21·7070
6·5	2.47. 6	2·984576	4·717068	21·8890
6·6	0.40. 7	3·084526	4·720072	21·9753

$\beta = -0{\cdot}6$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 43 . 31	·099834	·004994	0·0001
0·2	11 . 25 . 30	·198674	·019904	0·0012
0·3	17 . 4 . 24	·295556	·044517	0·0062
0·4	22 . 38 . 45	·389568	·078487	0·0189
0·5	28 . 7 . 6	·479873	·121350	0·0447
0·6	33 . 28 . 5	·565734	·172543	0·0890
0·7	38 . 40 . 23	·646522	·231419	0·1573
0·8	43 . 42 . 47	·721731	·297277	0·2546
0·9	48 . 34 . 10	·790978	·369380	0·3846
1·0	53 . 13 . 27	·854009	·446979	0·5499
1·1	57 . 39 . 42	·910690	·529333	0·7518
1·2	61 . 52 . 2	·961005	·615727	0·9899
1·3	65 . 49 . 37	1·005040	·705487	1·2628
1·4	69 . 31 . 44	1·042979	·797992	1·5679
1·5	72 . 57 . 43	1·075088	·892681	1·9019
1·6	76 . 6 . 55	1·101704	·989061	2·2609
1·7	78 . 58 . 46	1·123223	1·086707	2·6409
1·8	81 . 32 . 43	1·140092	1·185266	3·0377
1·9	83 . 48 . 14	1·152797	1·284449	3·4475
2·0	85 . 44 . 51	1·161856	1·384033	3·8667
2·1	87 . 22 . 5	1·167812	1·483852	4·2924
2·2	88 . 39 . 33	1·171230	1·583792	4·7220
2·3	89 . 36 . 50	1·172688	1·683780	5·1535
2·4	90 . 13 . 39	1·172776	1·783779	5·5856
2·5	90 . 29 . 46	1·172094	1·883777	6·0175
2·6	90 . 25 . 3	1·171246	1·983773	6·4488
2·7	89 . 59 . 31	1·170839	2·083772	6·8795
2·8	89 . 13 . 24	1·171474	2·183769	7·3103
2·9	88 . 7 . 4	1·173746	2·283742	7·7421
3·0	86 . 41 . 11	1·178232	2·383639	8·1759
3·1	84 . 56 . 37	1·185485	2·483371	8·6134
3·2	82 . 54 . 31	1·196025	2·582809	9·0561
3·3	80 . 36 . 16	1·210325	2·681774	9·5059
3·4	78 . 3 . 25	1·228805	2·780044	9·9648
3·5	75 . 17 . 41	1·251821	2·877349	10·4348
3·6	72 . 20 . 52	1·279657	2·973385	10·9179
3·7	69 . 14 . 46	1·312526	3·067816	11·4159
3·8	66 . 1 . 7	1·350560	3·160286	11·9306
3·9	62 . 41 . 32	1·393822	3·250429	12·4634

$\beta = -0{\cdot}6$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
4·0	59 . 17 . 28	1·442300	3·337875	13·0155
4·1	55 . 50 . 12	1·495923	3·422265	13·5873
4·2	52 . 20 . 49	1·554559	3·503250	14·1787
4·3	48 . 50 . 13	1·618032	3·580504	14·7890
4·4	45 . 19 . 10	1·686121	3·653721	15·4164
4·5	41 . 48 . 18	1·758573	3·722623	16·0581
4·6	38 . 18 . 7	1·835107	3·786962	16·7102
4·7	34 . 49 . 5	1·915420	3·846516	17·3676
4·8	31 . 21 . 35	1·999192	3·901099	18·0240
4·9	27 . 56 . 0	2·086091	3·950552	18·6717
5·0	24 . 32 . 41	2·175776	3·994752	19·3017
5·1	21 . 12 . 0	2·267903	4·033608	19·9037
5·2	17 . 54 . 18	2·362126	4·067063	20·4664
5·3	14 . 39 . 57	2·458103	4·095093	20·9772
5·4	11 . 29 . 22	2·555499	4·117708	21·4230
5·5	8 . 22 . 55	2·653989	4·134950	21·7898
5·6	5 . 21 . 3	2·753262	4·146893	22·0633
5·7	+2 . 24 . 10	2·853023	4·153640	22·2292
5·8	−0 . 27 . 19	2·952999	4·155326	22·2731

$\beta = -0{\cdot}7$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 43 . 28	·099834	·004994	0·0001
0·2	11 . 25 . 9	·198675	·019899	0·0012
0·3	17 . 3 . 15	·295562	·044492	0·0061
0·4	22 . 36 . 2	·389592	·078412	0·0189
0·5	28 . 1 . 51	·479947	·121173	0·0446
0·6	33 . 19 . 5	·565913	·172191	0·0888
0·7	38 . 26 . 14	·646897	·230798	0·1569
0·8	43 . 21 . 56	·722438	·296276	0·2537
0·9	48 . 4 . 51	·792208	·367876	0·3831
1·0	52 . 33 . 49	·856012	·444843	0·5478
1·1	56 . 47 . 44	·913783	·526440	0·7489
1·2	60 . 45 . 40	·965574	·611960	0·9866
1·3	64 . 26 . 42	1·011544	·700748	1·2595
1·4	67 . 50 . 3	1·051949	·792206	1·5657
1·5	70 . 55 . 0	1·087127	·885801	1·9024
1·6	73 . 40 . 56	1·117483	·981972	2·2664
1·7	76 . 7 . 16	1·143483	1·077625	2·6543
1·8	78 . 13 . 31	1·165634	1·175135	3·0629

$\beta = -0{\cdot}7$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
1·9	79 . 59 . 13	1·184483	1·273339	3·4891
2·0	81 . 24 . 2	1·200603	1·372028	3·9302
2·1	82 . 27 . 40	1·214589	1·471044	4·3839
2·2	83 . 9 . 55	1·227048	1·570264	4·8486
2·3	83 . 30 . 40	1·238597	1·669595	5·3229
2·4	83 . 29 . 56	1·249857	1·768959	5·8062
2·5	83 . 7 . 54	1·261446	1·868285	6·2981
2·6	82 . 24 . 51	1·273076	1·967496	6·7989
2·7	81 . 21 . 18	1·288044	2·066500	7·3092
2·8	79 . 57 . 55	1·304227	2·165180	7·8299
2·9	78 . 15 . 33	1·323071	2·263385	8·3621
3·0	76 . 15 . 16	1·345087	2·360926	8·9072
3·1	73 . 58 . 17	1·370730	2·457573	9·4668
3·2	71 . 25 . 57	1·400437	2·553052	10·0424
3·3	68 . 39 . 44	1·434528	2·647052	10·6354
3·4	65 . 41 . 9	1·473291	2·739221	11·2471
3·5	62 . 31 . 43	1·516933	2·829181	11·8785
3·6	59 . 12 . 57	1·565583	2·916533	12·5300
3·7	55 . 46 . 19	1·619300	3·000863	13·2014
3·8	52 . 13 . 10	1·678063	3·081756	13·8917
3·9	48 . 34 . 47	1·741787	3·158801	14·5989
4·0	44 . 52 . 21	1·810317	3·231603	15·3198
4·1	41 . 6 . 37	1·883442	3·299787	16·0499
4·2	37 . 19 . 35	1·960897	3·363010	16·7832
4·3	33 . 31 . 11	2·042370	3·420962	17·5121
4·4	29 . 42 . 37	2·127514	3·473372	18·2272
4·5	25 . 54 . 42	2·215948	3·520017	18·9177
4·6	22 . 8 . 15	2·307271	3·560719	19·5710
4·7	18 . 24 . 3	2·401064	3·595350	20·1732
4·8	14 . 42 . 50	2·496903	3·623835	20·7092
4·9	11 . 5 . 21	2·594364	3·646149	21·1627
5·0	7 . 32 . 22	2·693032	3·662320	21·5169
5·1	4 . 4 . 35	2·792505	3·672424	21·7550
5·2	0 . 42 . 43	2·892404	3·676587	21·8597

$\beta = -0{\cdot}8$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 43 . 26	·099834	·004993	0·0001
0·2	11 . 24 . 48	·198676	·019894	0·0012
0·3	17 . 2 . 6	·295568	·044468	0·0061
0·4	22 . 33 . 20	·389617	·078337	0·0189
0·5	27 . 56 . 36	·480020	·120996	0·0445
0·6	33 . 10 . 6	·566091	·171838	0·0886
0·7	38 . 12 . 9	·647270	·230177	0·1564
0·8	43 . 1 . 10	·723141	·295275	0·2528
0·9	47 . 35 . 42	·793427	·366371	0·3817
1·0	51 . 54 . 28	·857992	·442704	0·5456
1·1	55 . 56 . 15	·916833	·523534	0·7460
1·2	59 . 40 . 1	·970070	·608165	0·9830
1·3	63 . 4 . 48	1·017931	·695950	1·2558
1·4	66 . 9 . 48	1·060741	·786310	1·5628
1·5	68 . 54 . 17	1·098904	·878731	1·9017
1·6	71 . 17 . 38	1·132892	·972771	2·2698
1·7	73 . 19 . 20	1·163229	1·068052	2·6646
1·8	74 . 58 . 56	1·190485	1·164263	3·0834
1·9	76 . 16 . 8	1·215257	1·261144	3·5240
2·0	77 . 10 . 42	1·238168	1·358482	3·9842
2·1	77 . 42 . 33	1·259854	1·456102	4·4628
2·2	77 . 51 . 42	1·280958	1·553850	4·9584
2·3	77 . 38 . 18	1·302122	1·651585	5·4705
2·4	77 . 2 . 43	1·323983	1·749165	5·9990
2·5	76 . 5 . 24	1·347162	1·846441	6·5440
2·6	74 . 47 . 3	1·372258	1·943239	7·1061
2·7	73 . 8 . 29	1·399837	2·039357	7·6860
2·8	71 . 10 . 44	1·430428	2·134558	8·2848
2·9	68 . 54 . 57	1·464511	2·228563	8·9032
3·0	66 . 22 . 26	1·502507	2·321055	9·5424
3·1	63 . 34 . 37	1·544771	2·411673	10·2030
3·2	60 . 32 . 56	1·591588	2·500024	10·8852
3·3	57 . 18 . 56	1·643162	2·585683	11·5887
3·4	53 . 54 . 8	1·699618	2·668205	12·3125
3·5	50 . 20 . 3	1·760995	2·747133	13·0544
3·6	46 . 38 . 9	1·827252	2·822009	13·8110
3·7	42 . 49 . 51	1·898268	2·892387	14·5777
3·8	38 . 56 . 32	1·973848	2·957839	15·3478
3·9	34 . 59 . 31	2·053726	3·017968	16·1133
4·0	31 . 0 . 3	2·137577	3·072419	16·8638
4·1	26 . 59 . 23	2·225024	3·120885	17·5876
4·2	22 . 58 . 42	2·315649	3·163111	18·2706
4·3	18 . 59 . 7	2·409001	3·198907	18·8974
4·4	15 . 1 . 47	2·504611	3·228145	19·4510
4·5	11 . 7 . 47	2·602000	3·250762	19·9134
4·6	7 . 18 . 11	2·700692	3·266764	20·2659
4·7	+3 . 34 . 0	2·800226	3·276219	20·4897
4·8	−0 . 3 . 43	2·900164	3·279260	20·5664

$\beta = -0{\cdot}9$				
$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5.43.23	·099834	·004993	0·0001
0·2	11.24.28	·198677	·019889	0·0012
0·3	17. 0.57	·295574	·044443	0·0061
0·4	22.30.37	·389641	·078262	0·0189
0·5	27.51.22	·480093	·120820	0·0444
0·6	33. 1. 9	·566268	·171487	0·0884
0·7	37.58. 7	·647641	·229558	0·1559
0·8	42.40.30	·723837	·294276	0·2519
0·9	47. 6.45	·794632	·364867	0·3802
1·0	51.15.25	·859947	·440562	0·5433
1·1	55. 5.14	·919839	·520619	0·7429
1·2	58.35. 4	·974493	·604344	0·9792
1·3	61.43.57	1·024202	·691099	1·2518
1·4	64.31. 1	1·069354	·780314	1·5592
1·5	66.55.34	1·110418	·871486	1·8997
1·6	68.57. 2	1·147924	·964180	2·2712
1·7	70.34.56	1·182454	1·058026	2·6717
1·8	71.48.57	1·214627	1·152707	3·0992
1·9	72.38.55	1·245087	1·247954	3·5519
2·0	73. 4.45	1·274495	1·343532	4·0285
2·1	73. 6.35	1·303519	1·439227	4·5280
2·2	72.44.41	1·332823	1·534837	5·0499
2·3	71.59.28	1·363060	1·630155	5·5939
2·4	70.51.35	1·394863	1·724962	6·1602
2·5	69.21.48	1·428832	1·819012	6·7490
2·6	67.31. 7	1·465530	1·912031	7·3608
2·7	65.20.39	1·505470	2·003702	7·9960
2·8	62.51.43	1·549103	2·093672	8·6551
2·9	60. 5.43	1·596812	2·181546	9·3378
3·0	57. 4.10	1·648903	2·266894	10·0435
3·1	53.48.38	1·705596	2·349254	10·7710
3·2	50.20.46	1·767023	2·428145	11·5177
3·3	46.42.11	1·833223	2·503072	12·2799
3·4	42.54.33	1·904146	2·573544	13·0525
3·5	38.59.30	1·979648	2·639084	13·8283
3·6	34.58.36	2·059505	2·699241	14·5985
3·7	30.53.28	2·143412	2·753603	15·3521
3·8	26.45.37	2·231000	2·801812	16·0759
3·9	22.36.33	2·321842	2·843566	16·7548

$\beta = -0{\cdot}9$				
$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
4·0	18.27.46	2·415467	2·878636	17·3722
4·1	14.20.42	2·511378	2·906865	17·9095
4·2	10.16.45	2·609059	2·928175	18·3474
4·3	6.17.17	2·707998	2·942567	18·6660
4·4	+2.23.38	2·807693	2·950119	18·8455
4·5	−1.22.57	2·907671	2·950983	18·8667

$\beta = -1{\cdot}0$				
$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5.43.21	·099834	·004993	0·0001
0·2	11.24. 7	·198677	·019884	0·0012
0·3	16.59.48	·295580	·044419	0·0061
0·4	22.27.56	·389666	·078187	0·0188
0·5	27.46. 8	·480167	·120643	0·0444
0·6	32.52.14	·566444	·171136	0·0881
0·7	37.44. 8	·648009	·228939	0·1555
0·8	42.19.57	·724528	·293277	0·2510
0·9	46.37.59	·795826	·363362	0·3786
1·0	50.36.41	·861878	·438416	0·5411
1·1	54.14.42	·922803	·517693	0·7397
1·2	57.30.50	·978844	·600498	0·9753
1·3	60.24. 7	1·030357	·686196	1·2473
1·4	62.53.40	1·077790	·774222	1·5549
1·5	64.58.50	1·121669	·864075	1·8965
1·6	66.39. 5	1·162579	·955320	2·2707
1·7	67.54. 2	1·201151	1·047579	2·6757
1·8	68.43.31	1·238047	1·140523	3·1101
1·9	69. 7.29	1·273948	1·233856	3·5727
2·0	69. 6.[illegible]5	1·309543	1·327307	4·0627
2·1	68.39.39	1·345517	1·420612	4·5792
2·2	67.48.42	1·382541	1·513505	5·1221
2·3	66.33.56	1·421258	1·605704	5·6913
2·4	64.56.16	1·462276	1·696900	6·2866
2·5	62.56.48	1·506155	1·786753	6·9082
2·6	60.36.47	1·553395	1·874884	7·5559
2·7	57.57.39	1·604426	1·960873	8·2290
2·8	55. 0.58	1·659597	2·044263	8·9264
2·9	51.48.32	1·719167	2·124568	9·6460
3·0	48.21.36	1·783300	2·201275	10·3846

$\beta = -1·0$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
3·1	44 . 42 . 27	1·852061	2·273859	11·1375
3·2	40 . 52 . 45	1·925412	2·341800	11·8983
3·3	36 . 54 . 21	2·003218	2·404588	12·6589
3·4	32 . 49 . 5	2·085245	2·461748	13·4086
3·5	28 . 38 . 47	2·171177	2·512848	14·1350
3·6	24 . 25 . 16	2·260622	2·557515	14·8233
3·7	20 . 10 . 19	2·353123	2·595448	15·4566
3·8	15 . 55 . 42	2·448181	2·626423	16·0166
3·9	11 . 43 . 7	2·545264	2·650304	16·4834
4·0	7 . 34 . 16	2·643831	2·667043	16·8364
4·1	3 . 30 . 46	2·743344	2·676683	17·0552
4·2	− 0 . 25 . 49	2·843289	2·679353	

$\beta = -1·5$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 43 . 8	·099834	·004991	0·0001
0·2	11 . 22 . 25	·198681	·019860	0·0012
0·3	16 . 54 . 4	·295609	·044298	0·0061
0·4	22 . 14 . 28	·389787	·077813	0·0187
0·5	27 . 20 . 10	·480526	·119763	0·0440
0·6	32 . 7 . 59	·567311	·169387	0·0871
0·7	36 . 35 . 2	·649812	·225854	0·1531
0·8	40 . 38 . 44	·727894	·288295	0·2466
0·9	44 . 16 . 50	·801609	·355843	0·3710
1·0	47 . 27 . 26	·871181	·427656	0·5292
1·1	50 . 8 . 57	·736986	·502942	0·7228
1·2	52 . 20 . 8	·999526	·580964	0·9529
1·3	54 . 0 . 7	1·059409	·661047	1·2198
1·4	55 . 8 . 18	1·117316	·742573	1·5233
1·5	55 . 44 . 27	1·173979	·824970	1·8632
1·6	55 . 48 . 38	1·230159	·907697	2·2388
1·7	55 . 21 . 17	1·286618	·990234	2·6494
1·8	54 . 23 . 9	1·344100	1·072060	3·0942
1·9	52 . 55 . 18	1·403308	1·152645	3·5718
2·0	50 . 59 . 8	1·464883	1·231433	4·0807
2·1	48 . 36 . 19	1·529382	1·307843	4·6185
2·2	45 . 48 . 52	1·597265	1·381259	5·1819
2·3	42 . 39 . 1	1·668872	1·451044	5·7662
2·4	39 . 9 . 12	1·744414	1·516545	6·3651
2·5	35 . 22 . 7	1·823965	1·577109	6·9703
2·6	31 . 20 . 34	1·907458	1·632107	7·5711
2·7	27 . 7 . 29	1·994690	1·680955	8·1546
2·8	22 . 45 . 55	2·085332	1·723135	8·7054
2·9	18 . 18 . 55	2·178948	1·758221	9·2057
3·0	13 . 49 . 35	2·275015	1·785899	9·6361
3·1	9 . 20 . 58	2·372953	1·805975	9·9758
3·2	4 . 56 . 3	2·472154	1·818392	10·2038
3·3	0 . 37 . 44	2·572013	1·823224	10·2994

$\beta = -2·0$

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 42 . 55	·099834	·004990	0·0001
0·2	11 . 20 . 42	·198685	·019835	0·0012
0·3	16 . 48 . 22	·295639	·044176	0·0061
0·4	22 . 1 . 6	·389907	·077440	0·0186
0·5	26 . 54 . 28	·480881	·118885	0·0436
0·6	31 . 24 . 23	·568158	·167646	0·0860
0·7	35 . 27 . 18	·651557	·222786	0·1508
0·8	39 . 0 . 5	·731122	·283336	0·2420
0·9	42 . 0 . 11	·807098	·348339	0·3631
1·0	44 . 25 . 33	·879913	·416869	0·5166
1·1	46 . 14 . 41	·950142	·488052	0·7041
1·2	47 . 26 . 37	1·018473	·561062	0·9265
1·3	48 . 0 . 56	1·085668	·635122	1·1842
1·4	47 . 57 . 44	1·152528	·709484	1·4768
1·5	47 . 17 . 39	1·219857	·783422	1·8037
1·6	46 . 1 . 51	1·288423	·856211	2·1633
1·7	44 . 11 . 57	1·358932	·927117	2·5535
1·8	41 . 50 . 8	1·431991	·995388	2·9709
1·9	38 . 58 . 59	1·508082	1·060257	3·4110
2·0	35 . 41 . 32	1·587543	1·120948	3·8675
2·1	32 . 1 . 11	1·670544	1·176691	4·3318
2·2	28 . 1 . 38	1·757087	1·226753	4·7932
2·3	23 . 46 . 52	1·847006	1·270458	5·2384
2·4	19 . 21 . 2	1·939974	1·307226	5·6519
2·5	14 . 48 . 24	2·035536	1·336597	6·0157
2·6	10 . 13 . 17	2·133134	1·358260	6·3106
2·7	5 . 39 . 58	2·232150	1·372067	6·5163
2·8	1 . 12 . 36	2·331946	1·378044	6·6132

β = −2·5

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 42 . 42	·099834	·004988	0·0001
0·2	11 . 19 . 0	·198689	·019810	0·0012
0·3	16 . 42 . 40	·295668	·044054	0·0061
0·4	21 . 47 . 49	·390026	·077068	0·0185
0·5	26 . 29 . 1	·481229	·118012	0·0432
0·6	30 . 41 . 26	·568983	·165917	0·0850
0·7	34 . 20 . 53	·653244	·219737	0·1484
0·8	37 . 23 . 57	·734212	·278404	0·2374
0·9	39 . 47 . 55	·812298	·340862	0·3550
1·0	41 . 30 . 51	·888093	·406089	0·5034
1·1	42 . 31 . 36	·962313	·473104	0·6838
1·2	42 . 49 . 48	1·035760	·540968	0·8967
1·3	42 . 25 . 50	1·109265	·608769	1·1417
1·4	41 . 20 . 49	1·183643	·675609	1·4177
1·5	39 . 36 . 39	1·259645	·740593	1·7222
1·6	37 . 15 . 54	1·337915	·802822	2·0519
1·7	34 . 21 . 49	1·418953	·861394	2·4013
1·8	30 . 58 . 15	1·503087	·915418	2·7632
1·9	27 . 9 . 35	1·590451	·964038	3·1282
2·0	23 . 0 . 38	1·680981	1·006464	3·4843
2·1	18 . 36 . 35	1·774423	1·042010	3·8170
2·2	14 . 2 . 50	1·870360	1·070133	4·1097
2·3	9 . 24 . 53	1·968244	1·090463	4·3442
2·4	4 . 48 . 14	2·067449	1·102826	4·5016
2·5	0 . 18 . 16	2·167325	1·107258	4·5631

β = −3·0

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 42 . 29	·099834	·004986	0·0001
0·2	11 . 17 . 18	·198693	·019786	0·0012
0·3	16 . 37 . 0	·295697	·043933	0·0060
0·4	21 . 34 . 37	·390144	·076698	0·0184
0·5	26 . 3 . 50	·481571	·117144	0·0428
0·6	29 . 59 . 6	·569787	·164197	0·0839
0·7	33 . 15 . 48	·654875	·216708	0·1461
0·8	35 . 50 . 18	·737170	·273505	0·2328
0·9	37 . 39 . 58	·817223	·333427	0·3467
1·0	38 . 43 . 12	·895745	·395347	0·4896
1·1	38 . 59 . 28	·973546	·458171	0·6621
1·2	38 . 29 . 14	1·051477	·520835	0·8640
1·3	37 . 13 . 59	1·130361	·582291	1·0937
1·4	35 . 16 . 13	1·210941	·641502	1·3486
1·5	32 . 39 . 19	1·293823	·697437	1·6240
1·6	29 . 27 . 34	1·379437	·749087	1·9136
1·7	25 . 45 . 59	1·468001	·795487	2·2087
1·8	21 . 40 . 15	1·559514	·835750	2·4980
1·9	17 . 16 . 33	1·653759	·869111	2·7680
2·0	12 . 41 . 25	1·750328	·894977	3·0028
2·1	8 . 1 . 35	1·848671	·912956	3·1850
2·2	+3 . 23 . 42	1·948148	·922891	3·2967
2·3	−1 . 5 . 42	2·048103	·924869	3·3206

β = −3·5

$\frac{s}{b}$	ϕ (° ′ ″)	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
0·1	5 . 42 . 16	·099834	·004985	0·0001
0·2	11 . 15 . 36	·198697	·019761	0·0012
0·3	16 . 31 . 22	·295725	·043812	0·0060
0·4	21 . 21 . 31	·390259	·076329	0·0183
0·5	25 . 38 . 54	·481907	·116280	0·0424
0·6	29 . 17 . 23	·570571	·162489	0·0829
0·7	32 . 12 . 1	·656450	·213701	0·1437
0·8	34 . 19 . 6	·740000	·268640	0·2281
0·9	35 . 36 . 15	·821882	·326041	0·3382
1·0	36 . 2 . 27	·902892	·384669	0·4754
1·1	35 . 37 . 57	·983890	·443315	0·6394
1·2	34 . 24 . 22	1·065717	·500795	0·8290
1·3	32 . 24 . 32	1·149131	·555941	1·0414
1·4	29 . 42 . 30	1·234737	·607610	1·2718
1·5	26 . 23 . 23	1·322940	·654700	1·5135
1·6	22 . 33 . 17	1·413911	·696178	1·7571
1·7	18 . 19 . 4	1·507579	·731133	1·9909
1·8	13 . 48 . 12	1·603642	·758821	2·2008
1·9	9 . 8 . 28	1·701615	·778716	2·3709
2·0	+4 . 27 . 49	1·800884	·790551	2·4842
2·1	−0 . 6 . 0	1·900786	·794331	2·5241

$\beta = -4.0$					$\beta = -4.0$				
$\frac{s}{b}$	ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$	$\frac{s}{b}$	ϕ	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{V}{b^3}$
	° ′ ″					° ′ ″			
0·1	5 . 42 . 4	·099834	·004983	0·0001	1·0	33 . 28 . 25	·909560	·374082	0·4608
0·2	11 . 13 . 54	·198701	·019737	0·0012	1·1	32 . 26 . 47	·993395	·428593	0·6158
0·3	16 . 25 . 44	·295754	·043692	0·0060	1·2	30 . 34 . 42	1·078583	·480959	0·7923
0·4	21 . 8 . 30	·390374	·075961	0·0182	1·3	27 . 56 . 35	1·165764	·529924	0·9858
0·5	25 . 14 . 14	·482236	·115420	0·0420	1·4	24 . 38 . 10	1·255369	·574290	1·1898
0·6	28 . 36 . 18	·571336	·160791	0·0818	1·5	20 . 46 . 27	1·347574	·612947	1·3952
0·7	31 . 9 . 31	·657973	·210715	0·1413	1·6	16 . 29 . 22	1·442295	·644937	1·5902
0·8	32 . 50 . 18	·742707	·263813	0·2233	1·7	11 . 55 . 31	1·539203	·669502	1·7612
0·9	33 . 36 . 42	·826286	·318716	0·3296	1·8	7 . 13 . 56	1·637780	·686145	1·8925
					1·9	2 . 33 . 44	1·737388	·694666	1·9680

V

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
0·0	·2[illegible]882	·03407	·50000	·13397	·70711	·29289	·86603	·50000	1·00000	1·00000
0·1	·25860	·03403	·49838	·13330	·70232	·28972	·85673	·49078	·98421	·96321
[illegible]·2	·25838	·03399	·49679	·13264	·69770	·28666	·84794	·48217	·96976	·93113
0·3	·25816	·03395	·49522	·13200	·69324	·28372	·83962	·47412	·95648	·90285
0·4	·25795	·03390	·49368	·13137	·68894	·28090	·83174	·46657	·94422	·87764
0·5	·25774	·03386	·49217	·13075	·68478	·27819	·82427	·45946	·93283	·85491
0·6	·25752	·03381	·49068	·13015	·68075	·27558	·81715	·45275	·92217	·83423
0·7	·25730	·03377	·48922	·12955	·67684	·27306	·81035	·44640	·91215	·81529
0·8	·25710	·03373	·48777	·12896	·67305	·27062	·80384	·44038	·90271	·79786
0·9	·25689	·03369	·48635	·12838	·66937	·26827	·79760	·43465	·89377	·78173
1·0	·25668	·03365	·48495	·12781	·66579	·26599	·79161	·42919	·88529	·76671
1·1	·25647	·03360	·48357	·12725	·66231	·26378	·78585	·42397	·87722	·75268
1·2	·25626	·03356	·48221	·12670	·65892	·26164	·78031	·41898	·86953	·73954
1·3	·25606	·03352	·48087	·12616	·65562	·25957	·77496	·41420	·86218	·72719
1·4	·25585	·03348	·47955	·12563	·65241	·25756	·76979	·40962	·85513	·71554
1·5	·25564	·03344	·47826	·12511	·64928	·25560	·76478	·40522	·84838	·70453
1·6	·25543	·03340	·47698	·12460	·64622	·25370	·75994	·40098	·84189	·69410
1·7	·25523	·03336	·47571	·12409	·64323	·25185	·75526	·39690	·83564	·68418
1·8	·25503	·03332	·47447	·12360	·64032	·25004	·75072	·39297	·82963	·67475
1·9	·25482	·03328	·47324	·12311	·63747	·24829	·74631	·38918	·82383	·66576
2·0	·25462	·03324	·47203	·12262	·63469	·24658	·74203	·38552	·81822	·65717
2·1	·25442	·03321	·47084	·12214	·63196	·24491	·73787	·38198	·81280	·64895
2·2	·25422	·03317	·46965	·12167	·62929	·24328	·73382	·37855	·80755	·64109
2·3	·25402	·03313	·46848	·12121	·62668	·24169	·72988	·37522	·80247	·63353
2·4	·25383	·03309	·46733	·12075	·62412	·24014	·72605	·37200	·79754	·62628
2·5	·25363	·03305	·46619	·12030	·62161	·23863	·72231	·36888	·79275	·61931
2·6	·25343	·03302	·46507	·11986	·61915	·23715	·71866	·36585	·78810	·61260
2·7	·25324	·03298	·46396	·11942	·61674	·23570	·71510	·36290	·78358	·60613
2·8	·25304	·03294	·46286	·11899	·61437	·23429	·71162	·36004	·77919	·59989
2·9	·25285	·03290	·46178	·11856	·61205	·23290	·70823	·35726	·77491	·59386
3·0	·25265	·03286	·46071	·11814	·60978	·23155	·70492	·35454	·77074	·58803
3·1	·25246	·03282	·45966	·11773	·60754	·23022	·70168	·35189	·76667	·58240
3·2	·25227	·03279	·45861	·11732	·60534	·22892	·69851	·34932	·76270	·57694
3·3	·25208	·03275	·45757	·11692	·60318	·22765	·69540	·34681	·75883	·57165
3·4	·25189	·03271	·45655	·11652	·60106	·22640	·69236	·34437	·75506	·56652
3·5	·25170	·03268	·45553	·11613	·59898	·22517	·68939	·34199	·75137	·56154
3·6	·25150	·03264	·45453	·11573	·59693	·22397	·68647	·33966	·74776	·55672
3·7	·25132	·03260	·45355	·11535	·59491	·22279	·68361	·33739	·74423	·55202
3·8	·25113	·03256	·45257	·11497	·59292	·22163	·68080	·33517	·74078	·54745

V

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
0·0	·8660	1·5000	·7071	1·7071	·6428	1·7660	·5736	1·8192	·5000	1·8660
0·1	·8641	1·4152	·7276	1·5931	·6718	1·6462	·6175	1·6862	·5580	1·7238
0·2	·8602	1·3467	·7380	1·5062	·6898	1·5513	·6413	1·5875	·5905	1·6198
0·3	·8549	1·2897	·7432	1·4355	·7005	1·4750	·6559	1·5090	·6103	1·5381
0·4	·8490	1·2411	·7457	1·3762	·7064	1·4122	·6652	1·4439	·6233	1·4707
0·5	·8431	1·1988	·7464	1·3253	·7095	1·3591	·6712	1·3885	·6320	1·4135
0·6	·8370	1·1614	·7458	1·2808	·7109	1·3129	·6749	1·3404	·6379	1·3639
0·7	·8309	1·1280	·7442	1·2414	·7111	1·2718	·6771	1·2979	·6420	1·3202
0·8	·8248	1·0979	·7421	1·2061	·7106	1·2350	·6781	1·2599	·6447	1·2812
0·9	·8189	1·0705	·7396	1·1742	·7095	1·2018	·6783	1·2257	·6464	1·2461
1·0	·8131	1·0453	·7369	1·1451	·7079	1·1716	·6779	1·1947	·6472	1·2143
1·1	·8075	1·0221	·7340	1·1185	·7060	1·1439	·6771	1·1663	·6474	1·1852
1·2	·8020	1·0008	·7309	1·0939	·7039	1·1185	·6759	1·1401	·6472	1·1584
1·3	·7966	·9809	·7277	1·0711	·7016	1·0950	·6744	1·1158	·6467	1·1335
1·4	·7914	·9623	·7245	1·0499	·6992	1·0731	·6728	1·0933	·6459	1·1105
1·5	·7863	·9449	·7213	1·0301	·6966	1·0526	·6711	1·0723	·6448	1·0890
1·6	·7814	·9285	·7181	1·0115	·6940	1·0335	·6692	1·0526	·6435	1·0689
1·7	·7766	·9131	·7148	·9941	·6914	1·0154	·6671	1·0341	·6421	1·0501
1·8	·7719	·8985	·7116	·9776	·6888	·9985	·6650	1·0167	·6406	1·0323
1·9	·7674	·8847	·7085	·9620	·6861	·9824	·6629	1·0003	·6390	1·0154
2·0	·76297	·87162	·70525	·94720	·68339	·96723	·66068	·98467	·63738	·99952
2·1	·75866	·85916	·70212	·93318	·68072	·95280	·65848	·96988	·63565	·98443
2·2	·75445	·84728	·69903	·91985	·67806	·93909	·65626	·95582	·63387	·97009
2·3	·75035	·83595	·69599	·90714	·67541	·92603	·65402	·94243	·63205	·95643
2·4	·74636	·82512	·69299	·89500	·67278	·91354	·65178	·92965	·63021	·94340
2·5	·74246	·81474	·69004	·88339	·67018	·90159	·64955	·91744	·62834	·93095
2·6	·73865	·80479	·68712	·87227	·66760	·89016	·64732	·90575	·62647	·91904
2·7	·73494	·79525	·68425	·86161	·66504	·87921	·64510	·89454	·62459	·90762
2·8	·73131	·78608	·68142	·85137	·66252	·86870	·64289	·88378	·62271	·89666
2·9	·72776	·77725	·67864	·84153	·66003	·85859	·64069	·87344	·62082	·88613
3·0	·72428	·76873	·67590	·83207	·65758	·84887	·63851	·86350	·61893	·87599
3·1	·72088	·76052	·67321	·82295	·65516	·83951	·63635	·85392	·61705	·86623
3·2	·71756	·75260	·67055	·81415	·65276	·83048	·63421	·84468	·61518	·85682
3·3	·71430	·74495	·66794	·80566	·65039	·82176	·63209	·83577	·61332	·84774
3·4	·71111	·73755	·66537	·79745	·64805	·81334	·62999	·82716	·61147	·83897
3·5	·70799	·73039	·66285	·78951	·64574	·80519	·62792	·81884	·60963	·83050
3·6	·70493	·72345	·66036	·78182	·64346	·79731	·62588	·81080	·60780	·82230
3·7	·70193	·71673	·65791	·77438	·64122	·78968	·62385	·80300	·60599	·81437
3·8	·69898	·71022	·65550	·76717	·63901	·78228	·62183	·79544	·60419	·80669

V

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
3·9	·25094	·03253	·45160	·11460	·59097	·22050	·67805	·33300	·73741	·54301
4·0	·25076	·03249	·45064	·11423	·58905	·21938	·67536	·33087	·73411	·53869
4·1	·25057	·03245	·44969	·11386	·58716	·21828	·67272	·32879	·73088	·53448
4·2	·25039	·03242	·44875	·11350	·58529	·21721	·67011	·32676	·72771	·53038
4·3	·25021	·03238	·44783	·11314	·58345	·21615	·66756	·32477	·72460	·52638
4·4	·25002	·03234	·44691	·11279	·58164	·21511	·66505	·32282	·72155	·52248
4·5	·24984	·03231	·44600	·11244	·57986	·21409	·66258	·32091	·71856	·51868
4·6	·24966	·03227	·44510	·11209	·57810	·21309	·66016	·31903	·71562	·51497
4·7	·24948	·03223	·44421	·11175	·57637	·21210	·65778	·31720	·71274	·51134
4·8	·24930	·03220	·44333	·11141	·57466	·21112	·65543	·31540	·70991	·50780
4·9	·24912	·03216	·44246	·11108	·57298	·21016	·65313	·31364	·70713	·50434
5·0	·24894	·03213	·44159	·11075	·57133	·20922	·65086	·31191	·70441	·50095
5·1	·24876	·03209	·44073	·11042	·56969	·20829	·64863	·31021	·70173	·49764
5·2	·24858	·03206	·43988	·11010	·56807	·20738	·64644	·30854	·69909	·49440
5·3	·24841	·03203	·43904	·10978	·56647	·20648	·64428	·30690	·69650	·49122
5·4	·24823	·03199	·43821	·10947	·56490	·20559	·64215	·30529	·69395	·48811
5·5	·24806	·03196	·43738	·10916	·56335	·20472	·64005	·30371	·69145	·48508
5·6	·24788	·03193	·43656	·10885	·56182	·20386	·63798	·30215	·68899	·48210
5·7	·24771	·03189	·43575	·10854	·56031	·20301	·63595	·30062	·68657	·47918
5·8	·24754	·03186	·43495	·10824	·55881	·20218	·63394	·29912	·68418	·47632
5·9	·24736	·03183	·43415	·10794	·55733	·20136	·63196	·29765	·68183	·47351
6·0	·24719	·03179	·43336	·10764	·55588	·20055	·63001	·29621	·67952	·47076
6·1	·24702	·03176	·43258	·10735	·55444	·19975	·62809	·29479	·67724	·46806
6·2	·24685	·03173	·43180	·10706	·55302	·19896	·62619	·29339	·67500	·46541
6·3	·24668	·03170	·43103	·10677	·55161	·19818	·62432	·29201	·67279	·46280
6·4	·24651	·03166	·43027	·10648	·55021	·19741	·62247	·29064	·67061	·46024
6·5	·24634	·03163	·42951	·10620	·54884	·19666	·62065	·28930	·66846	·45774
6·6	·24617	·03160	·42876	·10592	·54748	·19592	·61885	·28798	·66634	·45528
6·7	·24600	·03157	·42802	·10564	·54614	·19518	·61708	·28668	·66425	·45286
6·8	·24584	·03154	·42729	·10536	·54481	·19445	·61533	·28540	·66219	·45048
6·9	·24567	·03150	·42656	·10509	·54350	·19373	·61360	·28414	·66016	·44814
7·0	·24550	·03147	·42583	·10482	·54220	·19302	·61189	·28291	·65815	·44584
7·1	·24534	·03144	·42511	·10455	·54091	·19232	·61020	·28169	·65617	·44357
7·2	·24517	·03141	·42440	·10429	·53964	·19163	·60853	·28048	·65422	·44134
7·3	·24500	·03138	·42369	·10403	·53838	·19095	·60688	·27929	·65229	·43915
7·4	·24484	·03134	·42299	·10377	·53714	·19027	·60526	·27812	·65039	·43700
7·5	·24467	·03131	·42229	·10351	·53591	·18960	·60366	·27697	·64851	·43488
7·6	·24451	·03128	·42160	·10326	·53469	·18894	·60207	·27583	·64666	·43280
7·7	·24434	·03125	·42091	·10300	·53349	·18828	·60050	·27471	·64483	·43075

V

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+ 3·9	·69609	·70390	·65312	·76017	·63683	·77511	·61985	·78811	·60241	·79924
4·0	·69326	·69775	·65078	·75338	·63467	·76814	·61790	·78101	·60065	·79201
4·1	·69048	·69178	·64848	·74678	·63254	·76138	·61597	·77411	·59890	·78499
4·2	·68775	·68598	·64621	·74037	·63045	·75481	·61405	·76741	·59717	·77817
4·3	·68506	·68033	·64398	·73413	·62838	·74842	·61216	·76089	·59545	·77154
4·4	·68242	·67483	·64178	·72807	·62634	·74221	·61029	·75454	·59375	·76509
4·5	·67984	·66948	·63960	·72217	·62433	·73616	·60844	·74837	·59207	·75881
4·6	·67730	·66427	·63746	·71642	·62234	·73027	·60662	·74236	·59040	·75269
4·7	·67480	·65918	·63535	·71082	·62038	·72453	·60481	·73650	·58875	·74673
4·8	·67233	·65421	·63327	·70536	·61845	·71894	·60302	·73079	·58712	·74092
4·9	·66991	·64937	·63122	·70003	·61654	·71349	·60126	·72522	·58551	·73526
5·0	·66753	·64464	·62920	·69483	·61466	·70816	·59951	·71979	·58391	·72973
5·1	·66519	·64002	·62721	·68975	·61280	·70296	·59778	·71449	·58232	·72434
5·2	·66288	·63551	·62525	·68479	·61096	·69788	·59607	·70931	·58075	·71908
5·3	·66061	·63110	·62331	·67996	·60915	·69292	·59439	·70425	·57920	·71393
5·4	·65837	·62679	·62139	·67523	·60736	·68808	·59273	·69931	·57767	·70890
5·5	·65617	·62258	·61950	·67059	·60559	·68334	·59109	·69448	·57616	·70399
5·6	·65400	·61846	·61764	·66606	·60384	·67871	·58947	·68976	·57466	·69919
5·7	·65187	·61442	·61580	·66163	·60211	·67417	·58787	·68514	·57317	·69449
5·8	·64977	·61046	·61399	·65730	·60041	·66973	·58628	·68061	·57169	·68989
5·9	·64769	·60659	·61220	·65305	·59873	·66539	·58470	·67618	·57023	·68539
6·0	·64564	·60280	·61043	·64889	·59707	·66114	·58314	·67184	·56879	·68098
6·1	·64362	·59908	·60868	·64482	·59543	·65697	·58160	·66759	·56736	·67666
6·2	·64163	·59544	·60696	·64082	·59381	·65289	·58008	·66343	·56594	·67243
6·3	·63967	·59186	·60526	·63691	·59220	·64889	·57858	·65935	·56454	·66828
6·4	·63773	·58836	·60358	·63307	·59061	·64496	·57710	·65534	·56316	·66421
6·5	·63582	·58492	·60192	·62931	·58904	·64111	·57563	·65142	·56179	·66023
6·6	·63394	·58154	·60028	·62561	·58749	·63733	·57417	·64757	·56044	·65632
6·7	·63208	·57823	·59866	·62199	·58596	·63362	·57273	·64379	·55910	·65248
6·8	·63024	·57498	·59706	·61844	·58445	·62999	·57131	·64008	·55777	·64871
6·9	·62843	·57179	·59547	·61495	·58295	·62642	·56990	·63643	·55645	·64501
7·0	·62664	·56865	·59390	·61151	·58147	·62291	·56851	·63286	·55514	·64138
7·1	·62487	·56557	·59235	·60814	·58001	·61946	·56713	·62935	·55385	·63781
7·2	·62312	·56254	·59082	·60483	·57856	·61608	·56577	·62590	·55257	·63430
7·3	·62140	·55956	·58931	·60157	·57712	·61276	·56442	·62251	·55130	·63085
7·4	·61970	·55663	·58781	·59837	·57570	·60949	·56308	·61918	·55004	·62747
7·5	·61802	·55375	·58633	·59522	·57430	·60627	·56175	·61590	·54880	·62415
7·6	·61636	·55092	·58487	·59214	·57291	·60311	·56044	·61268	·54756	·62088
7·7	·61472	·54813	·58342	·58910	·57153	·60000	·55914	·60951	·54634	·61766

V

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
7·8	·24418	·03122	·42023	·10275	·53230	·18764	·59895	·27360	·64302	·42872
7·9	·24402	·03118	·41956	·10250	·53112	·18701	·59741	·27251	·64123	·42672
8·0	·24386	·03115	·41889	·10225	·52995	·18638	·59590	·27143	·63947	·42476
8·1	·24370	·03112	·41822	·10201	·52879	·18576	·59440	·27037	·63773	·42282
8·2	·24354	·03109	·41756	·10176	·52764	·18515	·59292	·26932	·63600	·42091
8·3	·24338	·03106	·41691	·10152	·52651	·18454	·59145	·26828	·63430	·41903
8·4	·24322	·03103	·41626	·10128	·52539	·18394	·59000	·26725	·63262	·41718
8·5	·24306	·03100	·41562	·10105	·52428	·18334	·58857	·26624	·63096	·41536
8·6	·24291	·03097	·41498	·10081	·52318	·18275	·58716	·26524	·62932	·41356
8·7	·24275	·03094	·41434	·10057	·52208	·18217	·58576	·26425	·62769	·41179
8·8	·24259	·03091	·41371	·10034	·52100	·18159	·58437	·26328	·62608	·41004
8·9	·24244	·03088	·41308	·10011	·51993	·18102	·58299	·26232	·62449	·40831
9·0	·24228	·03085	·41246	·09989	·51887	·18046	·58162	·26137	·62291	·40661
9·1	·24213	·03082	·41184	·09966	·51782	·17990	·58027	·26043	·62135	·40493
9·2	·24197	·03079	·41122	·09944	·51677	·17935	·57893	·25950	·61981	·40327
9·3	·24182	·03076	·41061	·09922	·51574	·17881	·57761	·25858	·61829	·40163
9·4	·24167	·03073	·41001	·09900	·51472	·17827	·57631	·25767	·61679	·40001
9·5	·24151	·03070	·40941	·09878	·51371	·17773	·57502	·25678	·61530	·39842
9·6	·24136	·03067	·40882	·09856	·51270	·17720	·57374	·25590	·61382	·39685
9·7	·24121	·03064	·40823	·09835	·51171	·17668	·57247	·25503	·61236	·39529
9·8	·24105	·03062	·40764	·09813	·51072	·17616	·57121	·25416	·61092	·39375
9·9	·24090	·03059	·40705	·09792	·50974	·17565	·56997	·25330	·60950	·39223
10·0	·24075	·03056	·40647	·09771	·50877	·17514	·56874	·25245	·60808	·39074
10·1	·24060	·03053	·40589	·09750	·50781	·17464	·56753	·25161	·60667	·38926
10·2	·24045	·03051	·40532	·09729	·50686	·17414	·56632	·25078	·60528	·38780
10·3	·24030	·03048	·40475	·09709	·50591	·17364	·56512	·24996	·60391	·38636
10·4	·24015	·03045	·40418	·09688	·50497	·17315	·56393	·24915	·60255	·38493
10·5	·24000	·03042	·40362	·09668	·50404	·17267	·56275	·24835	·60121	·38352
10·6	·23986	·03039	·40306	·09648	·50312	·17219	·56159	·24756	·59988	·38212
10·7	·23971	·03037	·40251	·09628	·50220	·17172	·56044	·24677	·59856	·38074
10·8	·23956	·03034	·40196	·09608	·50129	·17125	·55929	·24599	·59725	·37938
10·9	·23942	·03031	·40141	·09589	·50039	·17078	·55815	·24522	·59595	·37804
11·0	·23927	·03028	·40087	·09569	·49949	·17032	·55702	·24446	·59466	·37671
11·1	·23913	·03025	·40033	·09550	·49860	·16986	·55590	·24371	·59338	·37539
11·2	·23898	·03023	·39979	·09531	·49772	·16941	·55479	·24296	·59212	·37409
11·3	·23884	·03020	·39926	·09512	·49685	·16896	·55369	·24222	·59087	·37280
11·4	·23869	·03017	·39873	·09493	·49598	·16851	·55261	·24149	·58963	·37153
11·5	·23855	·03014	·39820	·09474	·49512	·16807	·55153	·24077	·58840	·37027
11·6	·23841	·03011	·39768	·09456	·49427	·16763	·55046	·24005	·58719	·36903

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
7·8	·61310	·54539	·58199	·58611	·57017	·59694	·55785	·60639	·54513	·61449
7·9	·61149	·54269	·58057	·58317	·56883	·59393	·55658	·60333	·54394	·61138
8·0	·60990	·54003	·57917	·58027	·56750	·59097	·55532	·60032	·54276	·60832
8·1	·60833	·53741	·57778	·57742	·56618	·58805	·55407	·59735	·54158	·60531
8·2	·60678	·53484	·57641	·57461	·56488	·58518	·55283	·59443	·54041	·60235
8·3	·60525	·53230	·57505	·57185	·56359	·58236	·55161	·59155	·53926	·59943
8·4	·60373	·52981	·57371	·56913	·56231	·57958	·55040	·58872	·53812	·59655
8·5	·60223	·52735	·57238	·56645	·56104	·57684	·54920	·58593	·53699	·59371
8·6	·60074	·52493	·57106	·56381	·55978	·57414	·54801	·58318	·53587	·59092
8·7	·59927	·52254	·56976	·56120	·55854	·57148	·54684	·58047	·53476	·58817
8·8	·59782	·52019	·56847	·55864	·55731	·56886	·54567	·57780	·53366	·58546
8·9	·59639	·51787	·56719	·55611	·55609	·56628	·54451	·57517	·53256	·58279
9·0	·59497	·51558	·56592	·55362	·55488	·56373	·54336	·57258	·53147	·58015
9·1	·59356	·51332	·56467	·55116	·55368	·56122	·54223	·57003	·53039	·57755
9·2	·59217	·51110	·56342	·54874	·55249	·55875	·54110	·56751	·52932	·57499
9·3	·59079	·50891	·56219	·54635	·55131	·55631	·53998	·56502	·52826	·57247
9·4	·58942	·50675	·56097	·54399	·55015	·55390	·53888	·56257	·52721	·56998
9·5	·58807	·50461	·55976	·54167	·54900	·55153	·53778	·56015	·52618	·56753
9·6	·58673	·50250	·55856	·53938	·54786	·54919	·53669	·55776	·52515	·56511
9·7	·58541	·50042	·55737	·53711	·54673	·54688	·53561	·55541	·52413	·56272
9·8	·58410	·49837	·55620	·53488	·54561	·54460	·53454	·55309	·52312	·56037
9·9	·58280	·49634	·55504	·53268	·54450	·54235	·53348	·55080	·52211	·55805
10·0	·58151	·49434	·55389	·53050	·54339	·54013	·53243	·54854	·52111	·55575
10·1	·58023	·49236	·55275	·52835	·54230	·53794	·53138	·54631	·52012	·55349
10·2	·57897	·49041	·55162	·52623	·54122	·53577	·53034	·54411	·51914	·55126
10·3	·57772	·48849	·55050	·52414	·54014	·53363	·52932	·54194	·51817	·54905
10·4	·57648	·48659	·54939	·52207	·53907	·53152	·52831	·53979	·51720	·54687
10·5	·57525	·48471	·54828	·52003	·53801	·52944	·52731	·53767	·51624	·54471
10·6	·57403	·48285	·54718	·51801	·53696	·52738	·52631	·53558	·51529	·54258
10·7	·57282	·48102	·54610	·51602	·53592	·52535	·52532	·53351	·51435	·54048
10·8	·57163	·47921	·54503	·51406	·53489	·52334	·52434	·53146	·51342	·53840
10·9	·57045	·47742	·54396	·51211	·53387	·52136	·52336	·52944	·51249	·53635
11·0	·56928	·47565	·54290	·51019	·53286	·51940	·52239	·52744	·51157	·53433
11·1	·56812	·47390	·54186	·50829	·53185	·51746	·52143	·52546	·51066	·53233
11·2	·56696	·47218	·54082	·50642	·53085	·51554	·52048	·52351	·50976	·53035
11·3	·56582	·47047	·53978	·50456	·52986	·51365	·51953	·52158	·50886	·52839
11·4	·56469	·46878	·53876	·50273	·52888	·51178	·51859	·51968	·50796	·52646
11·5	·56357	·46711	·53774	·50092	·52791	·50993	·51766	·51780	·50707	·52455
11·6	·56246	·46546	·53674	·49913	·52695	·50810	·51674	·51594	·50619	·52266

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
11·7	·23826	·03009	·39716	·09437	·49343	·16720	·54940	·23934	·58598	·36780
11·8	·23812	·03006	·39664	·09418	·49259	·16677	·54835	·23864	·58478	·36658
11·9	·23798	·03003	·39613	·09400	·49176	·16634	·54731	·23794	·58359	·36537
12·0	·23784	·03000	·39562	·09382	·49093	·16592	·54628	·23725	·58241	·36418
12·1	·23770	·02998	·39512	·09364	·49011	·16550	·54525	·23657	·58124	·36300
12·2	·23756	·02995	·39461	·09346	·48930	·16509	·54423	·23589	·58008	·36183
12·3	·23742	·02992	·39411	·09328	·48849	·16468	·54322	·23522	·57893	·36067
12·4	·23728	·02989	·39361	·09311	·48769	·16427	·54222	·23456	·57779	·35952
12·5	·23714	·02987	·39311	·09293	·48689	·16386	·54123	·23390	·57667	·35839
12·6	·23700	·02984	·39262	·09276	·48610	·16346	·54024	·23325	·57555	·35727
12·7	·23687	·02981	·39213	·09258	·48531	·16306	·53926	·23260	·57444	·35616
12·8	·23673	·02979	·39164	·09241	·48453	·16266	·53829	·23196	·57334	·35506
12·9	·23659	·02976	·39116	·09224	·48375	·16227	·53733	·23133	·57225	·35397
13·0	·23645	·02974	·39067	·09207	·48298	·16188	·53638	·23070	·57117	·35289
13·1	·23632	·02971	·39019	·09190	·48221	·16149	·53543	·23008	·57010	·35182
13·2	·23618	·02969	·38971	·09173	·48145	·16111	·53449	·22946	·56904	·35076
13·3	·23604	·02966	·38924	·09156	·48070	·16073	·53356	·22885	·56798	·34971
13·4	·23591	·02964	·38876	·09140	·47995	·16035	·53263	·22824	·56693	·34867
13·5	·23577	·02961	·38829	·09123	·47921	·15998	·53171	·22764	·56589	·34765
13·6	·23563	·02959	·38782	·09107	·47847	·15961	·53079	·22704	·56486	·34663
13·7	·23550	·02956	·38736	·09090	·47774	·15924	·52988	·22645	·56383	·34562
13·8	·23536	·02954	·38690	·09074	·47701	·15887	·52898	·22586	·56281	·34462
13·9	·23523	·02951	·38644	·09058	·47628	·15851	·52809	·22528	·56180	·34363
14·0	·23509	·02949	·38598	·09042	·47556	·15815	·52721	·22470	·56080	·34265
14·1	·23496	·02946	·38553	·09026	·47484	·15779	·52633	·22413	·55980	·34168
14·2	·23482	·02944	·38508	·09010	·47413	·15744	·52546	·22356	·55881	·34072
14·3	·23469	·02941	·38463	·08994	·47343	·15709	·52459	·22300	·55783	·33976
14·4	·23456	·02939	·38418	·08979	·47273	·15674	·52373	·22244	·55686	·33881
14·5	·23443	·02936	·38374	·08963	·47203	·15639	·52287	·22188	·55590	·33787
14·6	·23429	·02934	·38329	·08948	·47134	·15604	·52202	·22133	·55494	·33694
14·7	·23416	·02931	·38285	·08932	·47065	·15570	·52117	·22078	·55399	·33602
14·8	·23403	·02929	·38241	·08917	·46997	·15536	·52033	·22024	·55305	·33511
14·9	·23390	·02926	·38198	·08902	·46929	·15503	·51950	·21971	·55211	·33420
15·0	·23377	·02924	·38155	·08887	·46861	·15469	·51867	·21918	·55118	·33330
15·1	·23364	·02922	·38112	·08872	·46794	·15436	·51784	·21865	·55026	·33241
15·2	·23352	·02919	·38069	·08857	·46727	·15403	·51702	·21813	·54934	·33153
15·3	·23339	·02917	·38027	·08842	·46661	·15370	·51621	·21761	·54843	·33065
15·4	·23326	·02914	·37984	·08828	·46595	·15337	·51541	·21709	·54752	·32978
15·5	·23313	·02912	·37942	·08813	·46530	·15305	·51461	·21658	·54662	·32892

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+ 11·7	·56136	·46383	·53574	·49736	·52599	·50629	·51582	·51410	·50532	·52079
11·8	·56026	·46222	·53475	·49561	·52504	·50450	·51491	·51228	·50445	·51894
11·9	·55917	·46062	·53377	·49387	·52410	·50274	·51401	·51048	·50359	·51712
12·0	·55809	·45904	·53279	·49216	·52316	·50099	·51311	·50870	·50273	·51532
12·1	·55702	·45748	·53182	·49047	·52223	·49926	·51222	·50694	·50188	·51354
12·2	·55596	·45594	·53086	·48879	·52131	·49755	·51134	·50520	·50104	·51178
12·3	·55491	·45441	·52991	·48713	·52040	·49586	·51046	·50348	·50020	·51003
12·4	·55387	·45290	·52897	·48549	·51949	·49418	·50959	·50177	·49937	·50830
12·5	·55283	·45140	·52803	·48387	·51859	·49252	·50873	·50008	·49855	·50658
12·6	·55180	·44992	·52710	·48226	·51769	·49088	·50787	·49841	·49773	·50488
12·7	·55078	·44845	·52617	·48067	·51680	·48926	·50702	·49676	·49692	·50320
12·8	·54977	·44700	·52526	·47910	·51592	·48765	·50617	·49513	·49611	·50154
12·9	·54877	·44556	·52435	·47754	·51505	·48606	·50533	·49351	·49531	·49991
13·0	·54778	·44414	·52344	·47600	·51418	·48449	·50450	·49191	·49451	·49829
13·1	·54679	·44273	·52254	·47447	·51332	·48293	·50367	·49033	·49372	·49669
13·2	·54581	·44134	·52165	·47296	·51247	·48139	·50285	·48876	·49294	·49510
13·3	·54484	·43996	·52077	·47147	·51162	·47987	·50203	·48721	·49216	·49352
13·4	·54388	·43859	·51989	·46999	·51077	·47836	·50122	·48567	·49138	·49196
13·5	·54292	·43723	·51902	·46852	·50993	·47686	·50042	·48415	·49061	·49041
13·6	·54197	·43589	·51816	·46707	·50910	·47538	·49962	·48264	·48985	·48888
13·7	·54103	·43456	·51730	·46563	·50827	·47391	·49883	·48115	·48909	·48737
13·8	·54009	·43325	·51645	·46421	·50745	·47246	·49804	·47967	·48834	·48587
13·9	·53916	·43195	·51560	·46280	·50663	·47102	·49726	·47821	·48759	·48438
14·0	·53824	·43066	·51476	·46140	·50582	·46959	·49648	·47676	·48684	·48291
14·1	·53732	·42938	·51392	·46002	·50501	·46818	·49571	·47533	·48610	·48145
14·2	·53641	·42811	·51309	·45865	·50421	·46678	·49494	·47391	·48536	·48000
14·3	·53551	·42686	·51227	·45729	·50342	·46539	·49418	·47250	·48463	·47857
14·4	·53461	·42562	·51145	·45594	·50263	·46402	·49342	·47110	·48390	·47716
14·5	·53372	·42439	·51064	·45461	·50185	·46266	·49267	·46971	·48318	·47576
14·6	·53283	·42317	·50983	·45329	·50107	·46131	·49192	·46834	·48246	·47437
14·7	·53195	·42196	·50903	·45198	·50030	·45997	·49118	·46698	·48175	·47299
14·8	·53108	·42076	·50824	·45068	·49953	·45865	·49044	·46563	·48104	·47162
14·9	·53022	·41957	·50745	·44939	·49877	·45734	·48971	·46430	·48034	·47027
15·0	·52936	·41839	·50666	·44811	·49801	·45604	·48898	·46298	·47964	·46893
15·1	·52851	·41722	·50588	·44685	·49726	·45475	·48826	·46167	·47895	·46760
15·2	·52766	·41606	·50510	·44560	·49651	·45347	·48754	·46037	·47826	·46628
15·3	·52682	·41491	·50433	·44436	·49576	·45220	·48682	·45908	·47757	·46497
15·4	·52598	·41378	·50356	·44313	·49502	·45095	·48611	·45780	·47689	·46368
15·5	·52515	·41265	·50280	·44191	·49429	·44971	·48540	·45654	·47621	·46240

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
15·6	·23300	·02910	·37900	·08798	·46465	·15273	·51382	·21607	·54573	·32807
15·7	·23288	·02907	·37858	·08784	·46400	·15241	·51303	·21557	·54484	·32722
15·8	·23275	·02905	·37817	·08769	·46336	·15209	·51225	·21507	·54396	·32638
15·9	·23262	·02902	·37775	·08755	·46272	·15178	·51147	·21457	·54308	·32554
16·0	·23249	·02900	·37734	·08741	·46209	·15147	·51069	·21408	·54221	·32471
16·1	·23237	·02898	·37693	·08727	·46146	·15116	·50992	·21359	·54135	·32389
16·2	·23224	·02895	·37652	·08713	·46084	·15085	·50915	·21310	·54049	·32308
16·3	·23211	·02893	·37612	·08699	·46022	·15054	·50839	·21262	·53964	·32227
16·4	·23198	·02890	·37571	·08685	·45960	·15024	·50764	·21214	·53879	·32147
16·5	·23186	·02888	·37531	·08671	·45898	·14994	·50689	·21167	·53795	·32067
16·6	·23173	·02886	·37491	·08657	·45837	·14964	·50615	·21120	·53711	·31988
16·7	·23161	·02883	·37451	·08644	·45776	·14934	·50541	·21073	·53628	·31910
16·8	·23148	·02881	·37412	·08630	·45715	·14904	·50467	·21026	·53545	·31832
16·9	·23136	·02879	·37372	·08617	·45655	·14875	·50394	·20980	·53463	·31755
17·0	·23124	·02877	·37333	·08603	·45595	·14846	·50321	·20934	·53382	·31678
17·1	·23112	·02874	·37294	·08590	·45535	·14817	·50249	·20888	·53301	·31602
17·2	·23099	·02872	·37255	·08576	·45476	·14788	·50177	·20843	·53220	·31527
17·3	·23087	·02870	·37217	·08563	·45417	·14759	·50105	·20798	·53140	·31452
17·4	·23075	·02868	·37178	·08550	·45359	·14731	·50034	·20753	·53061	·31378
17·5	·23063	·02865	·37140	·08537	·45301	·14703	·49963	·20709	·52982	·31304
17·6	·23050	·02863	·37102	·08524	·45243	·14675	·49893	·20665	·52904	·31231
17·7	·23038	·02861	·37064	·08511	·45185	·14647	·49823	·20621	·52826	·31158
17·8	·23026	·02859	·37026	·08498	·45128	·14619	·49754	·20578	·52749	·31086
17·9	·23014	·02856	·36989	·08485	·45071	·14591	·49685	·20535	·52672	·31015
18·0	·23002	·02854	·36951	·08472	·45014	·14564	·49616	·20493	·52595	·30944
18·1	·22990	·02852	·36914	·08459	·44958	·14537	·49548	·20451	·52519	·30874
18·2	·22978	·02850	·36877	·08446	·44902	·14510	·49480	·20409	·52443	·30804
18·3	·22966	·02847	·36840	·08434	·44846	·14483	·49413	·20367	·52368	·30734
18·4	·22954	·02845	·36803	·08421	·44790	·14457	·49346	·20325	·52293	·30665
18·5	·22942	·02843	·36767	·08409	·44735	·14430	·49279	·20284	·52219	·30596
18·6	·22930	·02841	·36730	·08397	·44680	·14404	·49213	·20243	·52145	·30528
18.7	·22919	·02839	·36694	·08384	·44626	·14378	·49147	·20202	·52072	·30460
18·8	·22907	·02836	·36658	·08372	·44572	·14351	·49081	·20161	·51999	·30393
18·9	·22895	·02834	·36622	·08360	·44518	·14325	·49016	·20121	·51926	·30327
19·0	·22883	·02832	·36586	·08348	·44464	·14299	·48951	·20081	·51854	·30261
19·1	·22872	·02830	·36551	·08336	·44410	·14273	·48886	·20041	·51782	·30195
19·2	·22860	·02828	·36515	·08324	·44357	·14248	·48822	·20002	·51711	·30130
19·3	·22848	·02825	·36479	·08312	·44304	·14222	·48758	·19963	·51640	·30065
19·4	·22837	·02823	·36444	·08300	·44251	·14197	·48695	·19924	·51569	·30001

V

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+15·6	·52432	·41153	·50204	·44070	·49356	·44848	·48470	·45529	·47554	·46113
15·7	·52350	·41043	·50129	·43950	·49284	·44726	·48400	·45405	·47487	·45987
15·8	·52268	·40933	·50055	·43831	·49212	·44604	·48331	·45282	·47420	·45862
15·9	·52187	·40824	·49981	·43713	·49140	·44484	·48262	·45159	·47354	·45738
16·0	·52107	·40716	·49907	·43596	·49069	·44365	·48193	·45037	·47288	·45614
16·1	·52027	·40609	·49834	·43480	·48998	·44247	·48125	·44916	·47222	·45491
16·2	·51947	·40503	·49761	·43365	·48928	·44130	·48057	·44797	·47157	·45369
16·3	·51868	·40398	·49689	·43252	·48858	·44014	·47990	·44679	·47092	·45249
16·4	·51790	·40293	·49617	·43139	·48789	·43898	·47923	·44562	·47028	·45130
16·5	·51712	·40189	·49545	·43026	·48720	·43783	·47856	·44446	·46964	·45013
16·6	·51635	·40086	·49474	·42915	·48651	·43670	·47790	·44331	·46901	·44896
16·7	·51558	·39984	·49403	·42805	·48583	·43558	·47724	·44217	·46838	·44780
16·8	·51482	·39883	·49333	·42695	·48515	·43446	·47659	·44103	·46775	·44665
16·9	·51406	·39783	·49263	·42586	·48447	·43335	·47594	·43990	·46712	·44551
17·0	·51330	·39683	·49194	·42478	·48380	·43225	·47529	·43878	·46650	·44437
17·1	·51255	·39584	·49125	·42372	·48313	·43116	·47465	·43767	·46588	·44325
17·2	·51180	·39486	·49056	·42266	·48247	·43008	·47401	·43657	·46526	·44214
17·3	·51106	·39389	·48988	·42160	·48181	·42901	·47338	·43548	·46465	·44103
17·4	·51032	·39292	·48920	·42056	·48115	·42795	·47275	·43439	·46404	·43993
17·5	·50959	·39196	·48853	·41952	·48050	·42689	·47212	·43331	·46344	·43884
17·6	·50886	·39101	·48786	·41850	·47985	·42584	·47150	·43224	·46284	·43776
17·7	·50814	·39006	·48720	·41748	·47921	·42480	·47088	·43119	·46224	·43669
17·8	·50742	·38912	·48653	·41646	·47857	·42377	·47026	·43014	·46165	·43562
17·9	·50670	·38819	·48587	·41546	·47793	·42274	·46964	·42910	·46106	·43456
18·0	·50599	·38727	·48522	·41446	·47730	·42172	·46903	·42807	·46047	·43351
18·1	·50528	·38635	·48457	·41347	·47667	·42071	·46842	·42704	·45989	·43247
18·2	·50458	·38544	·48392	·41249	·47605	·41971	·46782	·42602	·45931	·43144
18·3	·50388	·38454	·48328	·41151	·47543	·41871	·46722	·42501	·45873	·43041
18·4	·50319	·38364	·48264	·41054	·47481	·41772	·46662	·42401	·45815	·42939
18·5	·50250	·38275	·48200	·40958	·47419	·41674	·46603	·42302	·45758	·42838
18·6	·50181	·38186	·48137	·40863	·47358	·41577	·46544	·42203	·45701	·42738
18·7	·50113	·38098	·48074	·40768	·47297	·41481	·46485	·42105	·45644	·42639
18·8	·50045	·38011	·48011	·40674	·47236	·41385	·46426	·42007	·45588	·42540
18·9	·49977	·37925	·47949	·40581	·47176	·41290	·46368	·41910	·45532	·42442
19·0	·49910	·37839	·47887	·40488	·47116	·41195	·46310	·41814	·45476	·42344
19·1	·49843	·37754	·47825	·40396	·47056	·41101	·46252	·41718	·45420	·42247
19·2	·49777	·37669	·47764	·40304	·46997	·41008	·46195	·41623	·45365	·42151
19·3	·49711	·37585	·47703	·40214	·46938	·40915	·46138	·41529	·45310	·42056
19·4	·49645	·37501	·47642	·40124	·46879	·40823	·46081	·41436	·45255	·41961

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
19·5	·22825	·02821	·36409	·08288	·44199	·14172	·48632	·19885	·51499	·29937
19·6	·22814	·02819	·36374	·08276	·44147	·14147	·48569	·19846	·51429	·29873
19·7	·22802	·02817	·36339	·08264	·44095	·14123	·48507	·19808	·51359	·29810
19·8	·22791	·02815	·36305	·08253	·44043	·14098	·48445	·19770	·51290	·29747
19·9	·22779	·02813	·36270	·08241	·43991	·14074	·48383	·19732	·51221	·29685
20·0	·22768	·02811	·36236	·08230	·43941	·14050	·48322	·19694	·51153	·29623
20·1	·22756	·02808	·36202	·08218	·43890	·14026	·48261	·19657	·51085	·29562
20·2	·22745	·02806	·36168	·08207	·43840	·14002	·48200	·19619	·51017	·29501
20·3	·22734	·02804	·36134	·08195	·43789	·13978	·48140	·19582	·50950	·29440
20·4	·22722	·02802	·36101	·08184	·43739	·13954	·48079	·19545	·50883	·29380
20·5	·22711	·02800	·36067	·08173	·43689	·13930	·48019	·19509	·50817	·29320
20·6	·22700	·02798	·36034	·08162	·43640	·13907	·47959	·19473	·50751	·29261
20·7	·22688	·02796	·36000	·08151	·43590	·13883	·47900	·19437	·50685	·29202
20·8	·22677	·02794	·35967	·08140	·43541	·13860	·47842	·19401	·50620	·29143
20·9	·22666	·02792	·35934	·08129	·43492	·13837	·47783	·19366	·50554	·29084
21·0	·22655	·02790	·35901	·08118	·43443	·13814	·47725	·19330	·50489	·29026
21·1	·22644	·02788	·35868	·08107	·43395	·13791	·47667	·19295	·50424	·28968
21·2	·22633	·02786	·35835	·08096	·43347	·13769	·47609	·19260	·50360	·28911
21·3	·22622	·02784	·35803	·08085	·43299	·13746	·47552	·19225	·50296	·28854
21·4	·22611	·02782	·35770	·08075	·43251	·13723	·47495	·19191	·50233	·28797
21·5	·22600	·02780	·35738	·08064	·43203	·13701	·47438	·19156	·50170	·28741
21·6	·22589	·02778	·35706	·08053	·43156	·13679	·47382	·19122	·50107	·28685
21·7	·22578	·02776	·35674	·08042	·43109	·13657	·47325	·19088	·50045	·28630
21·8	·22567	·02774	·35642	·08031	·43062	·13635	·47269	·19054	·49983	·28575
21·9	·22556	·02772	·35611	·08021	·43015	·13613	·47213	·19021	·49922	·28520
22·0	·22545	·02770	·35579	·08010	·42969	·13591	·47158	·18987	·49860	·28465
22·1	·22534	·02768	·35547	·08000	·42923	·13570	·47103	·18954	·49799	·28411
22·2	·22524	·02766	·35516	·07989	·42877	·13548	·47048	·18921	·49738	·28357
22·3	·22513	·02764	·35484	·07979	·42831	·13526	·46994	·18888	·49677	·28303
22·4	·22502	·02762	·35453	·07969	·42786	·13505	·46939	·18855	·49617	·28250
22·5	·22491	·02760	·35422	·07958	·42740	·13484	·46885	·18822	·49557	·28197
22·6	·22481	·02758	·35391	·07948	·42695	·13463	·46831	·18790	·49497	·28144
22·7	·22470	·02756	·35360	·07938	·42650	·13442	·46778	·18758	·49438	·28092
22·8	·22459	·02754	·35330	·07927	·42605	·13421	·46724	·18726	·49379	·28040
22·9	·22449	·02752	·35299	·07917	·42561	·13400	·46671	·18694	·49320	·27988
23·0	·22438	·02750	·35269	·07907	·42516	·13379	·46618	·18662	·49262	·27937
23·1	·22427	·02748	·35239	·07897	·42472	·13359	·46566	·18631	·49204	·27886
23·2	·22417	·02746	·35208	·07887	·42428	·13338	·46513	·18599	·49146	·27835
23·3	·22406	·02744	·35178	·07877	·42384	·13317	·46461	·18568	·49089	·27784

V

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
19·5	·49580	·37418	·47582	·40034	·46821	·40732	·46024	·41343	·45201	·41867
19·6	·49515	·37336	·47522	·39945	·46763	·40641	·45968	·41251	·45147	·41774
19·7	·49450	·37254	·47462	·39857	·46705	·40551	·45912	·41160	·45093	·41682
19·8	·49386	·37173	·47402	·39770	·46647	·40462	·45856	·41069	·45039	·41590
19·9	·49322	·37092	·47343	·39683	·46590	·40373	·45801	·40979	·44986	·41498
20·0	·49258	·37012	·47285	·39596	·46533	·40285	·45746	·40889	·44933	·41407
20·1	·49195	·36933	·47226	·39510	·46476	·40197	·45691	·40800	·44881	·41317
20·2	·49132	·36854	·47168	·39425	·46420	·40110	·45637	·40712	·44828	·41227
20·3	·49069	·36775	·47110	·39340	·46364	·40024	·45583	·40624	·44776	·41138
20·4	·49007	·36697	·47053	·39256	·46308	·39938	·45529	·40537	·44724	·41050
20·5	·48945	·36619	·46996	·39172	·46253	·39853	·45475	·40450	·44672	·40962
20·6	·48883	·36542	·46939	·39089	·46198	·39768	·45422	·40364	·44621	·40875
20·7	·48822	·36465	·46883	·39006	·46143	·39684	·45369	·40278	·44570	·40789
20·8	·48761	·36389	·46826	·38924	·46088	·39600	·45316	·40193	·44519	·40703
20·9	·48700	·36314	·46770	·38843	·46033	·39517	·45263	·40109	·44468	·40617
21·0	·48640	·36239	·46714	·38762	·45979	·39435	·45210	·40025	·44417	·40532
21·1	·48580	·36165	·46659	·38682	·45925	·39353	·45158	·39942	·44367	·40447
21·2	·48520	·36091	·46603	·38602	·45871	·39272	·45106	·39859	·44317	·40363
21·3	·48461	·36017	·46548	·38523	·45818	·39191	·45054	·39777	·44267	·40280
21·4	·48402	·35944	·46493	·38444	·45765	·39111	·45003	·39695	·44217	·40197
21·5	·48343	·35871	·46439	·38365	·45712	·39031	·44952	·39614	·44168	·40115
21·6	·48285	·35799	·46385	·38287	·45660	·38952	·44901	·39533	·44119	·40033
21·7	·48226	·35727	·46331	·38210	·45608	·38873	·44850	·39453	·44070	·39952
21·8	·48168	·35656	·46277	·38133	·45556	·38794	·44800	·39373	·44021	·39871
21·9	·48110	·35585	·46224	·38056	·45504	·38716	·44750	·39294	·43972	·39791
22·0	·48053	·35514	·46170	·37980	·45452	·38639	·44700	·39215	·43924	·39711
22·1	·47996	·35444	·46117	·37905	·45401	·38562	·44650	·39137	·43876	·39632
22·2	·47939	·35374	·46064	·37830	·45350	·38486	·44601	·39059	·43828	·39553
22·3	·47883	·35305	·46012	·37755	·45299	·38410	·44552	·38982	·43780	·39475
22·4	·47826	·45236	·45959	·37681	·45248	·38334	·44503	·38905	·43733	·39397
22·5	·47770	·35168	·45907	·37607	·45198	·38259	·44454	·38829	·43686	·39319
22·6	·47715	·35100	·45855	·37534	·45148	·38184	·44405	·38753	·43639	·39242
22·7	·47659	·35033	·45804	·37461	·45098	·38110	·44357	·38678	·43592	·39166
22·8	·47604	·34966	·45753	·37389	·45048	·38036	·44309	·38603	·43545	·39090
22·9	·47549	·34899	·45702	·37317	·44998	·37963	·44261	·38528	·43499	·39014
23·0	·47494	·34833	·45651	·37246	·44949	·37891	·44214	·38454	·43453	·38939
23·1	·47440	·34767	·45601	·37175	·44900	·37818	·44166	·38380	·43407	·38864
23·2	·47385	·34701	·45550	·37104	·44851	·37746	·44119	·38307	·43361	·38790
23·3	·47331	·34636	·45500	·37034	·44802	·37674	·44072	·38234	·43315	·38716

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
23·4	·22396	·02742	·35148	·07867	·42341	·13297	·46409	·18537	·49031	·27734
23·5	·22385	·02740	·35118	·07857	·42297	·13277	·46357	·18506	·48974	·27684
23·6	·22375	·02739	·35088	·07847	·42254	·13257	·46306	·18476	·48917	·27634
23·7	·22364	·02737	·35059	·07837	·42211	·13237	·46254	·18445	·48861	·27585
23·8	·22354	·02735	·35029	·07827	·42168	·13217	·46203	·18415	·48804	·27536
23·9	·22343	·02733	·35000	·07818	·42125	·13197	·46152	·18385	·48748	·27487
24·0	·22333	·02731	·34971	·07808	·42082	·13177	·46102	·18355	·48692	·27438
24·1	·22323	·02729	·34942	·07798	·42040	·13158	·46051	·18325	·48636	·27389
24·2	·22312	·02727	·34913	·07789	·41998	·13138	·46001	·18296	·48581	·27341
24·3	·22302	·02725	·34884	·07779	·41956	·13118	·45951	·18266	·48526	·27293
24·4	·22291	·02723	·34855	·07769	·41914	·13099	·45902	·18236	·48471	·27245
24·5	·22281	·02721	·34826	·07760	·41872	·13080	·45853	·18207	·48417	·27198
24·6	·22271	·02720	·34797	·07750	·41830	·13061	·45803	·18178	·48363	·27151
24·7	·22260	·02718	·34769	·07741	·41789	·13042	·45754	·18149	·48309	·27104
24·8	·22250	·02716	·34740	·07732	·41748	·13023	·45705	·18120	·48255	·27058
24·9	·22240	·02714	·34712	·07722	·41707	·13004	·45657	·18092	·48202	·27012
25·0	·22230	·02712	·34684	·07713	·41666	·12985	·45609	·18063	·48148	·26966
25·1	·22220	·02711	·34656	·07704	·41626	·12967	·45561	·18035	·48095	·26921
25·2	·22210	·02709	·34628	·07694	·41585	·12948	·45513	·18007	·48042	·26875
25·3	·22200	·02707	·34600	·07685	·41544	·12929	·45466	·17979	·47990	·26830
25·4	·22190	·02705	·34572	·07676	·41504	·12911	·45418	·17951	·47937	·26785
25·5	·22180	·02703	·34544	·07667	·41464	·12892	·45370	·17923	·47885	·26740
25·6	·22170	·02702	·34516	·07658	·41424	·12874	·45323	·17896	·47833	·26695
25·7	·22160	·02700	·34489	·07649	·41385	·12856	·45276	·17868	·47782	·26651
25·8	·22150	·02698	·34461	·07640	·41345	·12838	·45230	·17840	·47730	·26607
25·9	·22140	·02696	·34434	·07631	·41306	·12820	·45183	·17813	·47679	·26563
26·0	·22130	·02694	·34407	·07622	·41267	·12802	·45137	·17786	·47628	·26519
26·1	·22120	·02693	·34380	·07613	·41228	·12785	·45091	·17759	·47577	·26476
26·2	·22111	·02691	·34353	·07604	·41189	·12767	·45045	·17732	·47527	·26433
26·3	·22101	·02689	·34326	·07596	·41151	·12749	·44999	·17706	·47476	·26390
26·4	·22091	·02687	·34299	·07587	·41112	·12732	·44954	·17679	·47426	·26347
26·5	·22081	·02685	·34272	·07578	·41073	·12714	·44908	·17652	·47376	·26304
26·6	·22071	·02684	·34245	·07569	·41035	·12696	·44863	·17626	·47327	·26262
26·7	·22062	·02682	·34219	·07561	·40997	·12679	·44818	·17600	·47277	·26220
26·8	·22052	·02680	·34192	·07552	·40959	·12661	·44773	·17574	·47228	·26178
26·9	·22042	·02678	·34166	·07543	·40921	·12644	·44729	·17548	·47179	·26136
27·0	·22032	·02676	·34140	·07535	·40883	·12627	·44684	·17522	·47130	·26094
27·1	·22022	·02675	·34113	·07526	·40846	·12610	·44640	·17496	·47082	·26053
27·2	·22013	·02673	·34087	·07518	·40808	·12593	·44596	·17471	·47033	·26012

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
23·4	·47277	·34571	·45450	·36964	·44754	·37603	·44025	·38162	·43270	·38643
23·5	·47224	·34507	·45401	·36895	·44706	·37532	·43978	·38090	·43225	·38570
23·6	·47171	·34443	·45351	·36826	·44658	·37462	·43932	·38019	·43180	·38498
23·7	·47118	·34380	·45302	·36757	·44610	·37392	·43886	·37948	·43135	·38426
23·8	·47066	·34317	·45253	·36689	·44562	·37322	·43840	·37877	·43091	·38354
23·9	·47013	·34254	·45205	·36621	·44515	·37253	·43794	·37807	·43047	·38283
24·0	·46961	·34191	·45156	·36554	·44468	·37185	·43748	·37737	·43003	·38212
24·1	·46909	·34129	·45108	·36487	·44421	·37117	·43703	·37668	·42959	·38141
24·2	·46858	·34067	·45060	·36420	·44374	·37049	·43658	·37599	·42915	·38071
24·3	·46806	·34005	·45012	·36354	·44327	·36981	·43613	·37530	·42871	·38001
24·4	·46755	·33944	·44964	·36288	·44281	·36914	·43568	·37462	·42828	·37932
24·5	·46704	·33883	·44916	·36223	·44235	·36847	·43523	·37394	·42785	·37864
24·6	·46653	·33823	·44869	·36158	·44189	·36781	·43478	·37326	·42742	·37796
24·7	·46603	·33763	·44822	·36093	·44143	·36715	·43434	·37259	·42699	·37728
24·8	·46552	·33703	·44775	·36029	·44098	·36649	·43390	·37192	·42656	·37660
24·9	·46502	·33644	·44729	·35965	·44053	·36584	·43346	·37126	·42614	·37593
25·0	·46452	·33585	·44682	·35901	·44008	·36519	·43302	·37060	·42571	·37526
25·1	·46402	·33526	·44636	·35838	·43963	·36454	·43258	·36994	·42529	·37460
25·2	·46353	·33467	·44590	·35775	·43918	·36390	·43215	·36929	·42487	·37394
25·3	·46303	·33409	·44544	·35712	·43873	·36326	·43172	·36864	·42445	·37328
25·4	·46254	·33351	·44498	·35650	·43829	·36262	·43129	·36800	·42403	·37262
25·5	·46205	·33293	·44452	·35587	·43785	·36199	·43086	·36736	·42362	·37197
25·6	·46157	·33236	·44407	·35525	·43741	·36136	·43043	·36672	·42321	·37132
25·7	·46108	·33179	·44362	·35464	·43697	·36074	·43000	·36609	·42280	·37067
25·8	·46060	·33122	·44317	·35403	·43653	·36012	·42958	·36546	·42239	·37003
25·9	·46012	·33066	·44273	·35343	·43610	·35950	·42916	·36483	·42198	·36939
26·0	·45964	·33010	·44228	·35282	·43567	·35888	·42874	·36420	·42157	·36876
26·1	·45917	·32954	·44184	·35222	·43524	·35827	·42832	·36358	·42117	·36813
26·2	·45869	·32899	·44140	·35162	·43481	·35766	·42790	·36296	·42076	·36750
26·3	·45822	·32844	·44096	·35103	·43438	·35705	·42748	·36234	·42036	·36688
26·4	·45775	·32789	·44052	·35044	·43395	·35645	·42707	·36173	·41996	·36626
26·5	·45728	·32734	·44008	·34985	·43353	·35585	·42666	·36112	·41956	·36564
26·6	·45681	·32680	·43965	·34927	·43311	·35525	·42625	·36051	·41916	·36502
26·7	·45635	·32626	·43922	·34868	·43269	·35466	·42584	·35991	·41877	·36441
26·8	·45588	·32572	·43879	·34810	·43227	·35407	·42543	·35931	·41837	·36380
26·9	·45542	·32519	·43836	·34752	·43185	·35348	·42503	·35871	·41798	·36320
27·0	·45496	·32465	·43793	·34695	·43143	·35290	·42463	·35812	·41759	·36260
27·1	·45450	·32412	·43750	·34638	·43102	·35232	·42423	·35753	·41720	·36200
27·2	·45405	·32359	·43708	·34581	·43061	·35174	·42383	·35694	·41682	·36141

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
27·3	·22003	·02671	·34061	·07510	·40771	·12576	·44552	·17445	·46985	·25971
27·4	·21993	·02669	·34035	·07501	·40734	·12559	·44508	·17419	·46937	·25930
27·5	·21983	·02667	·34009	·07493	·40697	·12542	·44464	·17394	·46889	·25889
27·6	·21974	·02666	·33983	·07485	·40660	·12525	·44421	·17369	·46841	·25849
27·7	·21964	·02664	·33958	·07476	·40624	·12509	·44378	·17344	·46794	·25809
27·8	·21954	·02662	·33932	·07468	·40587	·12492	·44335	·17319	·46746	·25769
27·9	·21945	·02660	·33906	·07460	·40550	·12475	·44292	·17294	·46699	·25729
28·0	·21935	·02659	·33881	·07451	·40514	·12459	·44249	·17269	·46652	·25690
28·1	·21926	·02657	·33855	·07443	·40478	·12443	·44207	·17245	·46605	·25651
28·2	·21916	·02655	·33830	·07435	·40442	·12426	·44164	·17220	·46559	·25612
28·3	·21907	·02654	·33805	·07426	·40407	·12410	·44122	·17195	·46512	·25573
28·4	·21897	·02652	·33780	·07418	·40371	·12394	·44080	·17171	·46466	·25534
28·5	·21888	·02650	·33755	·07410	·40335	·12378	·44038	·17147	·46420	·25495
28·6	·21879	·02649	·33730	·07402	·40300	·12362	·43997	·17123	·46374	·25457
28·7	·21869	·02647	·33705	·07394	·40264	·12346	·43955	·17099	·46329	·25419
28·8	·21860	·02645	·33681	·07386	·40229	·12330	·43913	·17075	·46283	·25381
28·9	·21850	·02644	·33656	·07378	·40194	·12314	·43872	·17052	·46238	·25343
29·0	·21841	·02642	·33631	·07370	·40159	·12298	·43831	·17028	·46193	·25305
29·1	·21832	·02640	·33607	·07362	·40125	·12282	·43790	·17004	·46148	·25268
29·2	·21822	·02639	·33582	·07354	·40090	·12267	·43750	·16981	·46104	·25230
29·3	·21813	·02637	·33558	·07347	·40055	·12251	·43709	·16957	·46059	·25193
29·4	·21803	·02636	·33533	·07339	·40021	·12235	·43668	·16934	·46015	·25156
29·5	·21794	·02634	·33509	·07331	·39986	·12220	·43628	·16911	·45971	·25119
29·6	·21785	·02632	·33485	·07323	·39952	·12204	·43588	·16888	·45927	·25082
29·7	·21775	·02631	·33460	·07315	·39918	·12189	·43548	·16865	·45883	·25046
29·8	·21766	·02629	·33436	·07308	·39884	·12174	·43508	·16843	·45839	·25009
29·9	·21757	·02628	·33412	·07300	·39850	·12159	·43469	·16820	·45796	·24973
30·0	·21748	·02626	·33388	·07292	·39816	·12144	·43429	·16797	·45752	·24937
30·1	·21738	·02624	·33364	·07285	·39783	·12129	·43389	·16775	·45709	·24901
30·2	·21729	·02623	·33340	·07277	·39749	·12114	·43350	·16752	·45666	·24865
30·3	·21720	·02621	·33317	·07269	·39715	·12099	·43311	·16730	·45623	·24829
30·4	·21711	·02620	·33293	·07262	·39682	·12084	·43272	·16707	·45580	·24794
30·5	·21702	·02618	·33269	·07254	·39649	·12069	·43233	·16685	·45537	·24759
30·6	·21693	·02616	·33246	·07247	·39616	·12054	·43195	·16663	·45495	·24724
30·7	·21684	·02615	·33222	·07239	·39583	·12040	·43156	·16641	·45453	·24689
30·8	·21675	·02613	·33199	·07232	·39551	·12025	·43118	·16620	·45411	·24654
30·9	·21666	·02612	·33176	·07224	·39518	·12010	·43079	·16598	·45369	·24620
31·0	·21657	·02610	·33153	·07217	·39485	·11996	·43041	·16576	·45327	·24585
31·1	·21648	·02608	·33130	·07209	·39453	·11981	·43003	·16555	·45285	·24551

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
27·3	·45359	·32307	·43665	·34525	·43020	·35117	·42343	·35636	·41643	·36082
27·4	·45314	·32255	·43623	·34468	·42979	·35060	·42303	·35578	·41604	·36023
27·5	·45269	·32203	·43581	·34412	·42938	·35003	·42264	·35520	·41566	·35964
27·6	·45224	·32152	·43539	·34356	·42897	·34946	·42224	·35463	·41528	·35906
27·7	·45180	·32100	·43498	·34301	·42856	·34890	·42185	·35406	·41490	·35848
27·8	·45135	·32049	·43456	·34246	·42816	·34834	·42146	·35349	·41452	·35790
27·9	·45091	·31998	·43415	·34192	·42776	·34778	·42107	·35292	·41414	·35732
28·0	·45047	·31947	·43374	·34137	·42736	·34722	·42068	·35235	·41376	·35675
28·1	·45003	·31897	·43333	·34083	·42696	·34667	·42029	·35179	·41339	·35618
28·2	·44969	·31847	·43292	·34029	·42656	·34612	·41990	·35123	·41301	·35561
28·3	·44916	·31797	·43252	·33975	·42616	·34557	·41952	·35067	·41264	·35505
28·4	·44872	·31748	·43211	·33922	·42577	·34502	·41914	·35012	·41227	·35449
28·5	·44829	·31698	·43171	·33868	·42538	·34448	·41876	·34957	·41190	·35393
28·6	·44786	·31649	·43131	·33815	·42499	·34394	·41838	·34902	·41153	·35338
28·7	·44743	·31600	·43091	·33762	·42460	·34340	·41800	·34848	·41116	·35283
28·8	·44700	·31551	·43051	·33710	·42421	·34287	·41762	·34794	·41080	·35228
28·9	·44658	·31502	·43012	·33658	·42382	·34234	·41724	·34740	·41044	·35173
29·0	·44615	·31453	·42972	·33606	·42344	·34182	·41687	·34686	·41008	·35118
29·1	·44573	·31406	·42933	·33554	·42306	·34129	·41650	·34633	·40971	·35064
29·2	·44531	·31358	·42893	·33503	·42268	·34076	·41613	·34580	·40935	·35010
29·3	·44489	·31311	·42854	·33451	·42230	·34024	·41576	·34527	·40899	·34956
29·4	·44447	·31263	·42815	·33400	·42192	·33972	·41539	·34474	·40864	·34902
29·5	·44405	·31216	·42776	·33349	·42154	·33920	·41502	·34421	·40828	·34849
29·6	·44364	·31169	·42737	·33299	·42116	·33869	·41465	·34369	·40792	·34796
29·7	·44323	·31122	·42699	·33249	·42078	·33818	·41428	·34317	·40757	·34743
29·8	·44282	·31076	·42660	·33199	·42041	·33767	·41392	·34265	·40722	·34691
29·9	·44241	·31029	·42622	·33149	·42004	·33716	·41356	·34213	·40687	·34639
30·0	·44200	·30983	·42584	·33099	·41967	·33666	·41320	·34162	·40652	·34587
30·1	·44159	·30937	·42546	·33050	·41930	·33616	·41284	·34111	·40617	·34536
30·2	·44119	·30892	·42508	·33001	·41893	·33566	·41248	·34060	·40582	·34484
30·3	·44078	·30846	·42470	·32952	·41856	·33516	·41212	·34009	·40548	·34433
30·4	·44038	·30801	·42433	·32904	·41819	·33466	·41176	·33959	·40513	·34382
30·5	·43998	·30756	·42395	·32855	·41783	·33417	·41141	·33909	·40479	·34331
30·6	·43958	·30711	·42358	·32807	·41747	·33368	·41106	·33859	·40445	·34281
30·7	·43918	·30667	·42321	·32759	·41711	·33319	·41071	·33809	·40411	·34230
30·8	·43879	·30622	·42284	·32711	·41675	·33270	·41036	·33760	·40377	·34180
30·9	·43839	·30578	·42247	·32664	·41639	·33222	·41001	·33711	·40343	·34130
31·0	·43800	·30534	·42210	·32616	·41603	·33173	·40966	·33662	·40309	·34080
31·1	·43761	·30490	·42173	·32569	·41567	·33125	·40931	·33613	·40275	·34031

β	15°		30°		45°		60°		90°	
+	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
31·2	·21639	·02607	·33107	·07202	·39420	·11967	·42965	·16533	·45244	·24517
31·3	·21630	·02605	·33084	·07194	·39388	·11953	·42928	·16512	·45202	·24483
31·4	·21621	·02604	·33061	·07187	·39356	·11938	·42890	·16490	·45161	·24449
31·5	·21612	·02602	·33038	·07180	·39324	·11924	·42852	·16469	·45120	·24415
31·6	·21604	·02600	·33015	·07172	·39292	·11910	·42815	·16448	·45079	·24381
31·7	·21595	·02599	·32992	·07165	·39260	·11896	·42778	·16427	·45038	·24347
31·8	·21586	·02597	·32970	·07158	·39229	·11882	·42741	·16406	·44998	·24314
31·9	·21577	·02596	·32947	·07151	·39197	·11868	·42704	·16385	·44957	·24281
32·0	·21568	·02594	·32924	·07144	·39165	·11854	·42667	·16364	·44917	·24248
32·1	·21560	·02592	·32902	·07136	·39134	·11840	·42631	·16343	·44877	·24215
32·2	·21551	·02591	·32879	·07129	·39102	·11826	·42594	·16323	·44837	·24182
32·3	·21542	·02589	·32857	·07122	·39071	·11813	·42557	·16302	·44797	·24149
32·4	·21533	·02588	·32834	·07115	·39040	·11799	·42521	·16281	·44757	·24117
32·5	·21524	·02586	·32812	·07108	·39009	·11785	·42485	·16261	·44718	·24085
32·6	·21516	·02584	·32790	·07101	·38978	·11772	·42449	·16240	·44678	·24053
32·7	·21507	·02583	·32768	·07094	·38947	·11758	·42413	·16220	·44639	·24021
32·8	·21498	·02581	·32746	·07087	·38917	·11744	·42378	·16200	·44600	·23989
32·9	·21490	·02580	·32724	·07080	·38886	·11731	·42342	·16180	·44561	·23957
33·0	·21481	·02578	·32702	·07073	·38855	·11717	·42306	·16160	·44522	·23925
33·1	·21472	·02576	·32680	·07066	·38825	·11704	·42271	·16140	·44483	·23893
33·2	·21464	·02575	·32658	·07060	·38794	·11690	·42235	·16120	·44444	·23861
33·3	·21455	·02573	·32636	·07053	·38764	·11677	·42200	·16101	·44406	·23830
33·4	·21447	·02572	·32615	·07046	·38734	·11664	·42165	·16081	·44367	·23799
33·5	·21438	·02570	·32593	·07039	·38704	·11650	·42130	·16061	·44329	·23768
33·6	·21429	·02569	·32571	·07032	·38674	·11637	·42095	·16042	·44291	·23737
33·7	·21421	·02567	·32550	·07026	·38645	·11624	·42060	·16022	·44253	·23706
33·8	·21412	·02566	·32528	·07019	·38615	·11611	·42026	·16003	·44215	·23675
33·9	·21404	·02564	·32507	·07012	·38585	·11598	·41991	·15983	·44177	·23645
34·0	·21395	·02563	·32486	·07005	·38556	·11585	·41956	·15964	·44140	·23615
34·1	·21387	·02561	·32464	·06998	·38526	·11572	·41922	·15945	·44102	·23585
34·2	·21378	·02560	·32443	·06992	·38497	·11559	·41887	·15925	·44065	·23555
34·3	·21370	·02558	·32422	·06985	·38468	·11547	·41853	·15906	·44028	·23525
34·4	·21361	·02557	·32401	·06978	·38439	·11534	·41819	·15887	·43991	·23495
34·5	·21353	·02555	·32380	·06972	·38410	·11521	·41785	·15868	·43954	·23465
34·6	·21345	·02554	·32359	·06965	·38381	·11509	·41752	·15849	·43917	·23435
34·7	·21336	·02552	·32338	·06958	·38352	·11496	·41718	·15831	·43880	·23405
34·8	·21328	·02551	·32318	·06952	·38324	·11483	·41685	·15812	·43844	·23375
34·9	·21319	·02549	·32297	·06945	·38295	·11471	·41651	·15793	·43807	·23346
35·0	·21311	·02548	·32276	·06939	·38266	·11458	·41618	·15775	·43771	·23317

V

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
31·2	·43722	·30447	·42137	·32522	·41531	·33078	·40896	·33564	·40242	·33981
31·3	·43683	·30403	·42100	·32475	·41496	·33030	·40862	·33516	·40208	·33932
31·4	·43645	·30360	·42064	·32429	·41461	·32983	·40828	·33468	·40175	·33883
31·5	·43606	·30317	·42028	·32382	·41425	·32936	·40794	·33420	·40142	·33835
31·6	·43568	·30274	·41992	·32336	·41390	·32889	·40760	·33372	·40109	·33787
31·7	·43529	·30231	·41956	·32290	·41355	·32842	·40726	·33324	·40076	·33739
31·8	·43491	·30189	·41920	·32244	·41320	·32795	·40692	·33277	·40043	·33691
31·9	·43453	·30146	·41884	·32199	·41285	·32748	·40658	·33230	·40010	·33644
32·0	·43415	·30104	·41849	·32154	·41251	·32702	·40625	·33183	·39977	·33596
32·1	·43377	·30062	·41813	·32109	·41216	·32656	·40591	·33136	·39945	·33549
32·2	·43340	·30020	·41778	·32064	·41182	·32610	·40558	·33089	·39912	·33502
32·3	·43302	·29979	·41743	·32019	·41147	·32564	·40525	·33043	·39879	·33455
32·4	·43264	·29937	·41708	·31975	·41113	·32518	·40492	·32997	·39847	·33409
32·5	·43227	·29896	·41673	·31930	·41079	·32473	·40459	·32951	·39815	·33362
32·6	·43190	·29855	·41638	·31886	·41045	·32428	·40426	·32905	·39783	·33315
32·7	·43153	·29814	·41603	·31842	·41011	·32383	·40393	·32859	·39751	·33269
32·8	·43116	·29773	·41569	·31798	·40977	·32338	·40360	·32814	·39720	·33223
32·9	·43079	·29733	·41535	·31755	·40944	·32294	·40327	·32769	·39688	·33178
33·0	·43043	·29692	·41501	·31712	·40911	·32250	·40295	·32724	·39656	·33133
33·1	·43006	·29652	·41466	·31669	·40877	·32206	·40262	·32679	·39625	·33087
33·2	·42970	·29612	·41432	·31626	·40844	·32162	·40230	·32634	·39593	·33042
33·3	·42934	·29572	·41398	·31583	·40811	·32118	·40198	·32590	·39562	·32997
33·4	·42898	·29532	·41364	·31541	·40778	·32075	·40166	·32546	·39531	·32953
33·5	·42862	·29492	·41330	·31498	·40745	·32032	·40134	·32502	·39500	·32908
33·6	·42826	·29453	·41297	·31456	·40712	·31989	·40102	·32458	·39469	·32864
33·7	·42790	·29414	·41263	·31414	·40679	·31946	·40070	·32415	·39438	·32820
33·8	·42755	·29375	·41230	·31372	·40647	·31903	·40038	·32371	·39408	·32776
33·9	·42719	·29336	·41196	·31330	·40615	·31860	·40006	·32328	·39377	·32732
34·0	·42683	·29297	·41163	·31288	·40583	·31818	·39975	·32285	·39346	·32688
34·1	·42648	·29259	·41130	·31246	·40551	·31776	·39943	·32242	·39316	·32645
34·2	·42613	·29220	·41097	·31205	·40519	·31734	·39912	·32199	·39285	·32601
34·3	·42578	·29182	·41064	·31163	·40487	·31692	·39881	·32157	·39254	·32558
34·4	·42543	·29144	·41031	·31122	·40455	·31650	·39850	·32114	·39224	·32515
34·5	·42508	·29106	·40998	·31082	·40423	·31608	·39819	·32072	·39194	·32472
34·6	·42473	·29068	·40965	·31041	·40391	·31567	·39788	·32030	·39164	·32430
34·7	·42439	·29031	·40933	·31001	·40359	·31526	·39757	·31988	·39134	·32387
34·8	·42404	·28993	·40900	·30961	·40327	·31485	·39726	·31947	·39105	·32345
34·9	·42369	·28955	·40868	·30921	·40296	·31444	·39695	·31905	·39075	·32303
35·0	·42335	·28918	·40836	·30881	·40265	·31403	·39665	·31864	·39045	·32261

β	15° $\frac{x}{b}$	15° $\frac{z}{b}$	30° $\frac{x}{b}$	30° $\frac{z}{b}$	45° $\frac{x}{b}$	45° $\frac{z}{b}$	60° $\frac{x}{b}$	60° $\frac{z}{b}$	90° $\frac{x}{b}$	90° $\frac{z}{b}$
+										
35·1	·21303	·02547	·32255	·06933	·38238	·11446	·41585	·15756	·43735	·23288
35·2	·21295	·02545	·32235	·06926	·38209	·11433	·41552	·15738	·43699	·23259
35·3	·21286	·02544	·32214	·06920	·38181	·11421	·41519	·15720	·43663	·23230
35·4	·21278	·02542	·32194	·06914	·38152	·11408	·41486	·15702	·43627	·23201
35·5	·21270	·02541	·32174	·06907	·38124	·11396	·41453	·15684	·43591	·23172
35·6	·21262	·02540	·32153	·06901	·38096	·11384	·41421	·15666	·43555	·23144
35·7	·21253	·02538	·32133	·06895	·38068	·11371	·41388	·15648	·43520	·23115
35·8	·21245	·02537	·32113	·06888	·38040	·11359	·41356	·15630	·43484	·23087
35·9	·21237	·02535	·32093	·06882	·38012	·11347	·41323	·15612	·43449	·23059
36·0	·21229	·02534	·32073	·06875	·37984	·11335	·41291	·15594	·43414	·23031
36·1	·21220	·02533	·32053	·06869	·37957	·11323	·41259	·15576	·43379	·23003
36·2	·21212	·02531	·32033	·06862	·37929	·11311	·41227	·15559	·43344	·22975
36·3	·21204	·02530	·32013	·06856	·37901	·11299	·41195	·15541	·43309	·22947
36·4	·21196	·02528	·31993	·06850	·37874	·11287	·41163	·15523	·43274	·22920
36·5	·21188	·02527	·31973	·06844	·37846	·11275	·41131	·15506	·43240	·22892
36·6	·21180	·02526	·31954	·06837	·37819	·11262	·41099	·15488	·43205	·22864
36·7	·21172	·02524	·31934	·06831	·37791	·11251	·41068	·15471	·43171	·22837
36·8	·21164	·02523	·31914	·06825	·37764	·11240	·41036	·15453	·43136	·22810
36·9	·21156	·02521	·31894	·06819	·37737	·11228	·41005	·15436	·43102	·22783
37·0	·21148	·02520	·31875	·06813	·37710	·11216	·40974	·15419	·43068	·22756
37·1	·21140	·02519	·31855	·06807	·37683	·11205	·40943	·15402	·43034	·22729
37·2	·21132	·02517	·31836	·06801	·37656	·11193	·40912	·15385	·43000	·22702
37·3	·21124	·02516	·31816	·06795	·37630	·11181	·40881	·15368	·42966	·22675
37·4	·21116	·02514	·31797	·06789	·37603	·11170	·40850	·15351	·42932	·22649
37·5	·21108	·02513	·31778	·06783	·37576	·11158	·40819	·15334	·42899	·22622
37·6	·21100	·02512	·31758	·06777	·37550	·11147	·40788	·15317	·42865	·22595
37·7	·21093	·02510	·31739	·06771	·37523	·11135	·40758	·15300	·42832	·22569
37·8	·21085	·02509	·31720	·06765	·37497	·11124	·40727	·15284	·42799	·22542
37·9	·21077	·02507	·31701	·06759	·37471	·11113	·40697	·15267	·42766	·22516
38·0	·21069	·02506	·31682	·06753	·37445	·11101	·40667	·15250	·42733	·22490
38·1	·21061	·02505	·31663	·06747	·37419	·11090	·40637	·15234	·42700	·22464
38·2	·21054	·02503	·31644	·06741	·37393	·11079	·40607	·15217	·42667	·22438
38·3	·21046	·02502	·31625	·06735	·37367	·11067	·40577	·15200	·42634	·22412
38·4	·21038	·02500	·31606	·06729	·37341	·11056	·40547	·15184	·42601	·22386
38·5	·21030	·02499	·31587	·06723	·37315	·11045	·40517	·15167	·42569	·22360
38·6	·21022	·02498	·31569	·06717	·37290	·11034	·40487	·15151	·42536	·22335
38·7	·21015	·02496	·31550	·06712	·37264	·11023	·40458	·15134	·42504	·22309
38·8	·21007	·02495	·31531	·06706	·37238	·11012	·40428	·15118	·42472	·22283
38·9	·20999	·02493	·31513	·06700	·37213	·11001	·40398	·15102	·42440	·22258

V

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
35·1	·42301	·28881	·40804	·30841	·40233	·31362	·39634	·31823	·39015	·32220
35·2	·42267	·28844	·40772	·30801	·40202	·31322	·39604	·31782	·38986	·32178
35·3	·42233	·28807	·40740	·30762	·40171	·31282	·39574	·31741	·38956	·32136
35·4	·42199	·28771	·40708	·30722	·40140	·31242	·39544	·31701	·38927	·32095
35·5	·42165	·28734	·40677	·30683	·40109	·31202	·39514	·31660	·38898	·32054
35·6	·42131	·28698	·40645	·30644	·40078	·31162	·39484	·31620	·38869	·32013
35·7	·42098	·28662	·40614	·30605	·40047	·31122	·39454	·31579	·38840	·31972
35·8	·42064	·28626	·40583	·30566	·40016	·31083	·39424	·31539	·38811	·31932
35·9	·42031	·28590	·40551	·30528	·39986	·31044	·39394	·31499	·38782	·31891
36·0	·41998	·28554	·40520	·30489	·39956	·31005	·39365	·31459	·38753	·31850
36·1	·41965	·28518	·40489	·30451	·39925	·30966	·39335	·31419	·38725	·31810
36·2	·41932	·28483	·40458	·30413	·39895	·30927	·39306	·31380	·38696	·31770
36·3	·41900	·28447	·40427	·30375	·39865	·30888	·39276	·31340	·38667	·31730
36·4	·41867	·28412	·40396	·30337	·39835	·30850	·39247	·31301	·38638	·31690
36·5	·41834	·28377	·40365	·30299	·39805	·30812	·39218	·31262	·38610	·31650
36·6	·41802	·28342	·40335	·30261	·39775	·30774	·39189	·31223	·38581	·31611
36·7	·41769	·28307	·40304	·30224	·39745	·30736	·39160	·31185	·38553	·31572
36·8	·41737	·28273	·40274	·30186	·39715	·30698	·39131	·31146	·38525	·31533
36·9	·41704	·28238	·40244	·30149	·39685	·30660	·39102	·31108	·38497	·31494
37·0	·41672	·28204	·40214	·30112	·39656	·30622	·39074	·31070	·38469	·31455
37·1	·41640	·28169	·40183	·30075	·39626	·30584	·39045	·31032	·38441	·31417
37·2	·41608	·28135	·40153	·30038	·39597	·30546	·39017	·30994	·38414	·31378
37·3	·41576	·28101	·40123	·30002	·39568	·30509	·38988	·30956	·38386	·31340
37·4	·41544	·28067	·40093	·29965	·39539	·30472	·38960	·30919	·38358	·31302
37·5	·41512	·28033	·40063	·29929	·39510	·30435	·38932	·30881	·38330	·31264
37·6	·41481	·27999	·40034	·29893	·39481	·30398	·38903	·30843	·38303	·31226
37·7	·41449	·27966	·40004	·29857	·39452	·30361	·38875	·30806	·38275	·31189
37·8	·41418	·27932	·39975	·29821	·39423	·30325	·38847	·30769	·38248	·31151
37·9	·41386	·27899	·39945	·29786	·39394	·30289	·38819	·30732	·38221	·31113
38·0	·41355	·27866	·39916	·29750	·39366	·30253	·38791	·30695	·38194	·31075
38·1	·41324	·27833	·39887	·29714	·39337	·30217	·38763	·30658	·38167	·31038
38·2	·41293	·27800	·39857	·29679	·39309	·30181	·38735	·30622	·38141	·31001
38·3	·41262	·27767	·39828	·29643	·39280	·30145	·38707	·30585	·38114	·30964
38·4	·41231	·27735	·39799	·29608	·39252	·30109	·38679	·30548	·38087	·30927
38·5	·41200	·27702	·39770	·29573	·39224	·30074	·38652	·30512	·38060	·30890
38·6	·41170	·27670	·39741	·29538	·39196	·30038	·38624	·30476	·38034	·30854
38·7	·41139	·27637	·39713	·29503	·39168	·30003	·38597	·30440	·38007	·30817
38·8	·41109	·27605	·39684	·29469	·39140	·29967	·38570	·30405	·37980	·30781
38·9	·41078	·27573	·39656	·29434	·39112	·29932	·38543	·30369	·37953	·30745

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
39·0	·20991	·02492	·31494	·06694	·37187	·10990	·40369	·15086	·42408	·22233
39·1	·20983	·02491	·31475	·06688	·37162	·10979	·40339	·15070	·42376	·22208
39·2	·20976	·02489	·31457	·06683	·37136	·10969	·40310	·15054	·42344	·22183
39·3	·20968	·02488	·31438	·06677	·37111	·10958	·40281	·15038	·42312	·22158
39·4	·20960	·02486	·31420	·06671	·37086	·10947	·40252	·15022	·42280	·22133
39·5	·20952	·02485	·31401	·06665	·37061	·10936	·40223	·15006	·42249	·22108
39·6	·20945	·02484	·31383	·06660	·37036	·10925	·40194	·14990	·42217	·22084
39·7	·20937	·02482	·31365	·06654	·37011	·10915	·40165	·14975	·42186	·22059
39·8	·20929	·02481	·31346	·06648	·36986	·10904	·40137	·14959	·42155	·22034
39·9	·20922	·02479	·31328	·06643	·36961	·10893	·40108	·14943	·42124	·22010
40·0	·20914	·02478	·31310	·06637	·36936	·10882	·40079	·14928	·42093	·21986
40·1	·20906	·02477	·31292	·06631	·36911	·10872	·40051	·14912	·42062	·21962
40·2	·20899	·02475	·31274	·06626	·36887	·10861	·40022	·14897	·42031	·21938
40·3	·20891	·02474	·31256	·06620	·36862	·10850	·39994	·14881	·42000	·21914
40·4	·20884	·02472	·31238	·06614	·36837	·10840	·39965	·14866	·41970	·21890
40·5	·20876	·02471	·31220	·06609	·36813	·10829	·39937	·14851	·41939	·21866
40·6	·20868	·02470	·31202	·06603	·36788	·10818	·39909	·14835	·41909	·21842
40·7	·20861	·02468	·31185	·06598	·36764	·10808	·39881	·14820	·41878	·21818
40·8	·20853	·02467	·31167	·06592	·36739	·10797	·39853	·14805	·41848	·21795
40·9	·20846	·02465	·31149	·06587	·36715	·10787	·39825	·14790	·41818	·21771
41·0	·20838	·02464	·31131	·06582	·36691	·10777	·39797	·14775	·41787	·21747
41·1	·20831	·02463	·31114	·06576	·36667	·10766	·39769	·14760	·41757	·21724
41·2	·20823	·02461	·31096	·06571	·36643	·10756	·39742	·14745	·41727	·21700
41·3	·20816	·02460	·31078	·06565	·36619	·10746	·39714	·14730	·41697	·21677
41·4	·20808	·02458	·31061	·06560	·36595	·10736	·39686	·14715	·41667	·21653
41·5	·20801	·02457	·31043	·06555	·36571	·10726	·39659	·14700	·41637	·21630
41·6	·20793	·02456	·31025	·06549	·36547	·10716	·39631	·14685	·41607	·21607
41·7	·20786	·02454	·31008	·06544	·36524	·10706	·39604	·14671	·41578	·21584
41·8	·20778	·02453	·30990	·06538	·36500	·10696	·39577	·14656	·41548	·21561
41·9	·20771	·02452	·30973	·06533	·36477	·10686	·39550	·14641	·41519	·21539
42·0	·20764	·02451	·30956	·06528	·36454	·10676	·39523	·14627	·41489	·21516
42·1	·20756	·02449	·30938	·06522	·36430	·10666	·39496	·14612	·41460	·21493
42·2	·20749	·02448	·30921	·06517	·36407	·10657	·39469	·14598	·41430	·21471
42·3	·20741	·02447	·30904	·06511	·36384	·10647	·39442	·14584	·41401	·21448
42·4	·20734	·02446	·30887	·06506	·36361	·10637	·39416	·14569	·41372	·21425
42·5	·20727	·02444	·30870	·06501	·36338	·10627	·39389	·14555	·41343	·21403
42·6	·20719	·02443	·30853	·06495	·36315	·10617	·39362	·14541	·41314	·21381
42·7	·20712	·02442	·30836	·06490	·36292	·10608	·39336	·14526	·41285	·21358
42·8	·20705	·02441	·30819	·06485	·36269	·10598	·39309	·14512	·41256	·21336

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
39·0	·41048	·27540	·39627	·29400	·39084	·29897	·38516	·30334	·37927	·30709
39·1	·41018	·27509	·39599	·29366	·39056	·29862	·38489	·30299	·37901	·30674
39·2	·40987	·27477	·39570	·29332	·39028	·29827	·38462	·30263	·37874	·30638
39·3	·40957	·27445	·39542	·29298	·39000	·29792	·38435	·30228	·37848	·30602
39·4	·40927	·27414	·39513	·29264	·38973	·29758	·38408	·30193	·37822	·30566
39·5	·40897	·27382	·39485	·29230	·38946	·29724	·38382	·30158	·37796	·30531
39·6	·40868	·27351	·39457	·29196	·38918	·29690	·38355	·30123	·37770	·30496
39·7	·40838	·27319	·39429	·29163	·38891	·29656	·38328	·30088	·37745	·30461
39·8	·40808	·27288	·39401	·29129	·38864	·29622	·38302	·30054	·37719	·30426
39·9	·40779	·27257	·39373	·29096	·38837	·26588	·38275	·30019	·37693	·30391
40·0	·40749	·27226	·39346	·29063	·38810	·29554	·38249	·29985	·37667	·30356
40·1	·40720	·27195	·39318	·29030	·38783	·29520	·38222	·29951	·37642	·30322
40·2	·40691	·27165	·39290	·28997	·38756	·29487	·38196	·29917	·37616	·30287
40·3	·40661	·27134	·39263	·28965	·38729	·29453	·38170	·29883	·37590	·30252
40·4	·40632	·27103	·39235	·28932	·38702	·29420	·38144	·29849	·37565	·30218
40·5	·40603	·27073	·39208	·28899	·38676	·29387	·38118	·29815	·37540	·30184
40·6	·40574	·27043	·39181	·28867	·38649	·29354	·38092	·29782	·37514	·30150
40·7	·40545	·27013	·39154	·28834	·38622	·29321	·38066	·29748	·37489	·30116
40·8	·40517	·26983	·39127	·28802	·38596	·29288	·38040	·29714	·37464	·30083
40·9	·40488	·26953	·39100	·28770	·38569	·29255	·38014	·29681	·37439	·30049
41·0	·40459	·26923	·39073	·28738	·38543	·29223	·37989	·29648	·37414	·30015
41·1	·40431	·26894	·39047	·28705	·38516	·29190	·37963	·29615	·37389	·29982
41·2	·40402	·26864	·39020	·28673	·38490	·29158	·37937	·29582	·37364	·29948
41·3	·40374	·26834	·38993	·28641	·38464	·29126	·37912	·29550	·37340	·29915
41·4	·40346	·26805	·38967	·28610	·38438	·29094	·37887	·29517	·37315	·29882
41·5	·40317	·26775	·38940	·28578	·38412	·29062	·37862	·29484	·37290	·29849
41·6	·40289	·26746	·38914	·28547	·38386	·29030	·37836	·29452	·37265	·29816
41·7	·40261	·26717	·38887	·28516	·38360	·28998	·37811	·29419	·37240	·29784
41·8	·40233	·26688	·38861	·28485	·38334	·28966	·37786	·29387	·37216	·29751
41·9	·40205	·26659	·38834	·28454	·38309	·28935	·37761	·29355	·37192	·29718
42·0	·40177	·26630	·38808	·28423	·38284	·28903	·37736	·29323	·37168	·29685
42·1	·40149	·26602	·38782	·28392	·38258	·28871	·37711	·29291	·37143	·29653
42·2	·40122	·26573	·38756	·28362	·38233	·28840	·37686	·29260	·37119	·29620
42·3	·40094	·26544	·38730	·28331	·38207	·28809	·37661	·29228	·37095	·29588
42·4	·40066	·26516	·38704	·28300	·38182	·28778	·37636	·29196	·37071	·29556
42·5	·40039	·26487	·38678	·28270	·38157	·28747	·37612	·29165	·37047	·29524
42·6	·40011	·26459	·38652	·28239	·38132	·28716	·37587	·29134	·37023	·29492
42·7	·39984	·26431	·38627	·28209	·38107	·28685	·37562	·29102	·37000	·29461
42·8	·39957	·26403	·38601	·28179	·38080	·28654	·37538	·29071	·36976	·29429

β	15°		30°		45°		60°		90°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+42·9	·20698	·02440	·30802	·06480	·36246	·10588	·39282	·14498	·41228	·21314
43·0	·20691	·02438	·30785	·06475	·36223	·10578	·39256	·14483	·41199	·21292
43·1	·20683	·02437	·30768	·06469	·36200	·10569	·39229	·14469	·41170	·21270
43·2	·20676	·02436	·30752	·06464	·36177	·10559	·39203	·14455	·41142	·21248
43·3	·20669	·02435	·30735	·06459	·36155	·10549	·39177	·14441	·41113	·21227
43·4	·20662	·02434	·30718	·06454	·36132	·10539	·39151	·14427	·41085	·21205
43·5	·20655	·02432	·30701	·06449	·36109	·10530	·39125	·14413	·41057	·21183
43·6	·20647	·02431	·30684	·06444	·36087	·10520	·39099	·14399	·41029	·21161
43·7	·20640	·02430	·30668	·06439	·36064	·10510	·39074	·14385	·41001	·21140
43·8	·20633	·02429	·30651	·06434	·36042	·10501	·39048	·14372	·40973	·21118
43·9	·20626	·02428	·30635	·06429	·36019	·10491	·39022	·14358	·40945	·21096
44·0	·20619	·02426	·30618	·06424	·35997	·10482	·38997	·14344	·40917	·21075
44·1	·20611	·02425	·30601	·06419	·35975	·10472	·38971	·14331	·40890	·21054
44·2	·20604	·02424	·30585	·06414	·35952	·10463	·38946	·14317	·40862	·21032
44·3	·20597	·02423	·30568	·06409	·35930	·10453	·38921	·14303	·40834	·21011
44·4	·20590	·02422	·30552	·06404	·35908	·10444	·38895	·14290	·40807	·20990
44·5	·20583	·02420	·30536	·06399	·35886	·10435	·38870	·14276	·40779	·20969
44·6	·20576	·02419	·30519	·06394	·35864	·10425	·38845	·14263	·40752	·20948
44·7	·20569	·02418	·30503	·06389	·35842	·10416	·38819	·14249	·40724	·20927
44·8	·20562	·02417	·30487	·06384	·35821	·10407	·38794	·14236	·40697	·20906
44·9	·20555	·02416	·30471	·06379	·35799	·10398	·38769	·14222	·40670	·20886
45·0	·20548	·02414	·30455	·06374	·35777	·10389	·38744	·14209	·40643	·20865
45·1	·20541	·02413	·30439	·06369	·35755	·10380	·38719	·14196	·40616	·20844
45·2	·20534	·02412	·30423	·06365	·35734	·10371	·38694	·14182	·40589	·20824
45·3	·20527	·02411	·30407	·06360	·35712	·10362	·38670	·14169	·40562	·20803
45·4	·20520	·02410	·30391	·06355	·35690	·10353	·38645	·14156	·40536	·20782
45·5	·20513	·02408	·30375	·06350	·35669	·10344	·38620	·14143	·40509	·20762
45·6	·20506	·02407	·30359	·06346	·35647	·10335	·38596	·14130	·40482	·20742
45·7	·20499	·02406	·30344	·06341	·35626	·10326	·38571	·14117	·40456	·20721
45·8	·20492	·02405	·30328	·06336	·35604	·10317	·38546	·14104	·40429	·20701
45·9	·20485	·02404	·30312	·06331	·35583	·10308	·38522	·14091	·40402	·20681
46·0	·20478	·02402	·30296	·06326	·35562	·10299	·38497	·14078	·40376	·20661
46·1	·20471	·02401	·30281	·06322	·35541	·10290	·38473	·14065	·40350	·20641
46·2	·20464	·02400	·30265	·06317	·35520	·10282	·38449	·14052	·40323	·20621
46·3	·20457	·02399	·30249	·06312	·35499	·10273	·38424	·14040	·40297	·20601
46·4	·20450	·02398	·30234	·06307	·35478	·10264	·38400	·14027	·40271	·20581
46·5	·20443	·02396	·30218	·06302	·35457	·10255	·38376	·14014	·40245	·20561
46·6	·20436	·02395	·30202	·06298	·35436	·10246	·38352	·14001	·40219	·20542
46·7	·20430	·02394	·30186	·06293	·35416	·10238	·38328	·13988	·40193	·20522

β	120°		135°		140°		145°		150°	
	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$	$\frac{x}{b}$	$\frac{z}{b}$
+										
42·9	·39930	·26375	·38575	·28149	·38057	·28624	·37514	·29040	·36952	·29397
43·0	·39903	·26346	·38550	·28119	·38032	·28593	·37490	·29008	·36929	·29366
43·1	·39876	·26320	·38524	·28089	·38007	·28562	·37465	·28978	·36905	·29335
43·2	·39849	·26292	·38499	·28059	·37982	·28532	·37441	·28947	·36881	·29304
43·3	·39823	·26264	·38473	·28029	·37957	·28502	·37417	·28917	·36857	·29273
43·4	·39796	·26237	·38448	·28000	·37932	·28472	·37393	·28886	·36834	·29242
43·5	·39769	·26209	·38423	·27970	·37908	·28442	·37369	·28855	·36811	·29211
43·6	·39743	·26181	·38398	·27941	·37883	·28412	·37345	·28825	·36787	·29180
43·7	·39716	·26154	·38373	·27911	·37859	·28382	·37321	·28795	·36764	·29150
43·8	·39689	·26126	·38348	·27882	·37834	·28352	·37297	·28764	·36741	·29119
43·9	·39663	·26099	·38323	·27853	·37810	·28323	·37273	·28734	·36718	·29088
44·0	·39636	·26072	·38298	·27824	·37786	·28293	·37250	·28704	·36695	·29058
44·1	·39610	·26045	·38274	·27795	·37761	·28263	·37226	·28674	·36672	·29028
44·2	·39584	·26018	·38249	·27766	·37737	·28234	·37202	·28644	·36649	·28998
44·3	·39557	·25992	·38224	·27738	·37713	·28205	·37179	·28614	·36626	·28968
44·4	·39531	·25965	·38200	·27709	·37689	·28176	·37155	·28585	·36603	·28938
44·5	·39505	·25938	·38175	·27680	·37665	·28147	·37132	·28555	·36580	·28908
44·6	·39479	·25912	·38151	·27652	·37641	·28118	·37109	·28525	·36558	·28879
44·7	·39453	·25885	·38126	·27623	·37617	·28089	·37086	·28496	·36535	·28849
44·8	·39427	·25859	·38102	·27595	·37593	·28060	·37062	·28467	·36512	·28819
44·9	·39401	·25833	·38077	·27567	·37569	·28031	·37039	·28438	·36489	·28789
45·0	·39376	·25807	·38053	·27539	·37546	·28002	·37016	·28409	·36467	·28760
45·1	·39350	·25780	·38029	·27511	·37522	·27974	·36993	·28380	·36444	·28731
45·2	·39325	·25754	·38005	·27483	·37498	·27946	·36970	·28351	·36421	·28701
45·3	·39299	·25728	·37981	·27455	·37475	·27917	·36947	·28322	·36399	·28672
45·4	·39274	·25702	·37957	·27427	·37451	·27889	·36924	·28293	·36377	·28643
45·5	·39249	·25676	·37933	·27399	·37428	·27861	·36901	·28265	·36355	·28614
45·6	·39223	·25650	·37909	·27372	·37405	·27833	·36878	·28236	·36333	·28585
45·7	·39198	·25625	·37885	·27344	·37381	·27805	·36856	·28207	·36311	·28557
45·8	·39173	·25599	·37861	·27317	·37358	·27777	·36833	·28179	·36289	·28528
45·9	·39148	·25574	·37838	·27289	·37335	·27749	·36810	·28151	·36267	·28499
46·0	·39123	·25549	·37814	·27262	·37312	·27721	·36788	·28123	·36245	·28470
46·1	·39098	·25523	·37790	·27235	·37289	·27693	·36765	·28095	·36223	·28442
46·2	·39073	·25498	·37767	·27208	·37266	·27666	·36743	·28067	·36202	·28413
46·3	·39048	·25473	·37743	·27181	·37243	·27638	·36720	·28039	·36180	·28384
46·4	·39023	·25448	·37720	·27154	·37220	·27610	·36698	·28011	·36158	·28356
46·5	·38999	·25423	·37697	·27127	·37197	·27583	·36676	·27983	·36136	·28328
46·6	·38974	·25398	·37673	·27100	·37174	·27556	·36654	·27956	·36114	·28300
46·7	·28950	·25373	·37650	·27074	·37152	·27529	·36632	·27928	·36093	·28272

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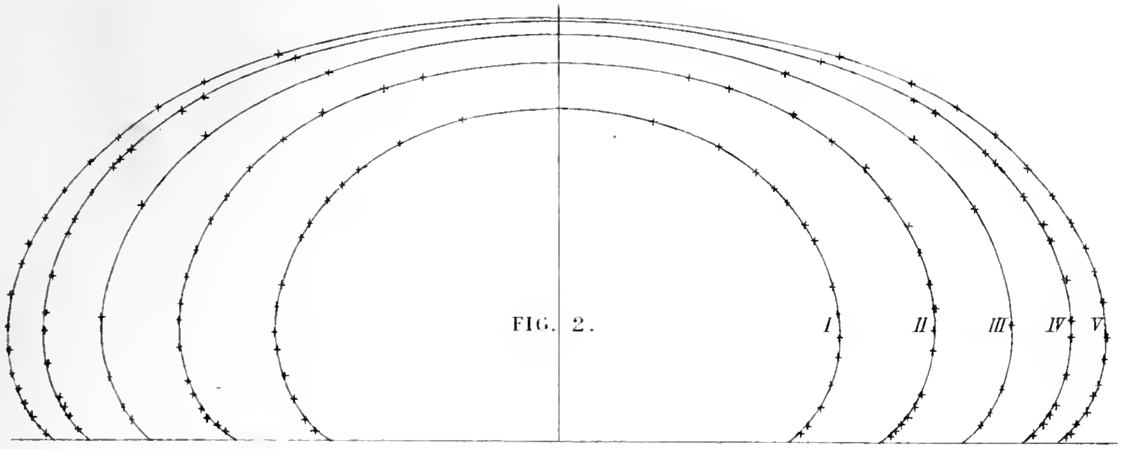

FIG. 2.

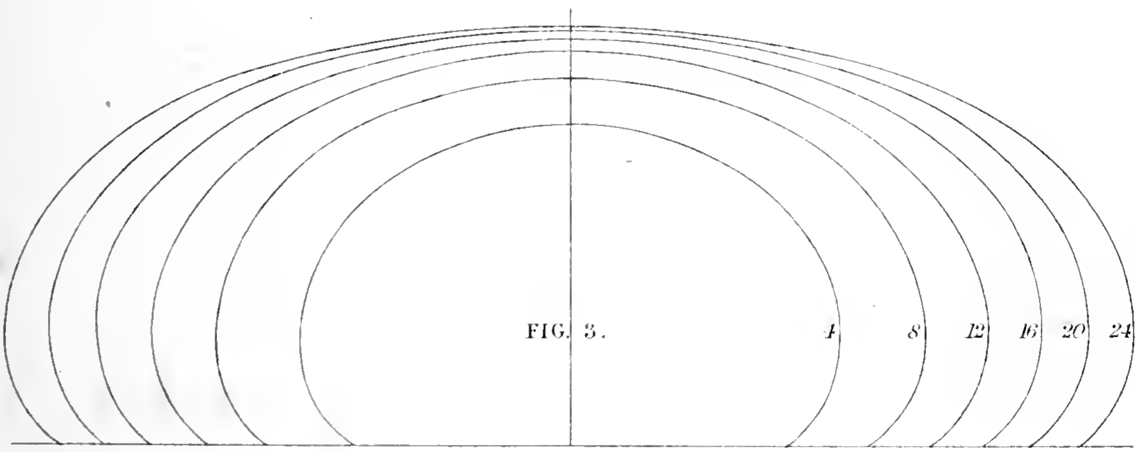

FIG. 3.

Scale.

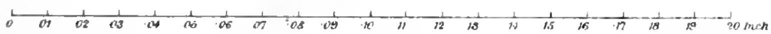

FIG. 1.

FIG. 4.